Market Ambiguity Attitude Restores the Risk-Return Tradeoff*

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Abstract

A positive relation between the conditional mean and conditional volatility of aggregate stock returns, while viewed as a fundamental law of finance, has been challenging to find empirically. We consider a representative agent asset pricing model with Knightian uncertainty and demonstrate that this risk-return tradeoff depends on the agent's ambiguity attitude (reflecting the agent's degree of optimism or pessimism). The model predicts the conditional equity premium is increasing in market volatility, but its slope flattens as market optimism rises. We develop a methodology to extract the representative agent's ambiguity attitude from our asset pricing model. Results validate our model predictions and document the significant in-sample and out-of-sample explanatory power of ambiguity attitude in explaining the risk-return tradeoff. In our sample, market volatility is not significant in forecasting returns. However, including the market ambiguity attitude leads to a significant positive relationship between volatility and future returns. Hence, our model and results identify market ambiguity attitude as a missing state variable that can explain why the literature has found it difficult to empirically validate the risk-return tradeoff.

JEL Classification: D8, D81, G12, G40, G41

Keywords: Ambiguity Attitude; Risk-Return Tradeoff; Equity Premium; Knightian Uncertainty

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1 Introduction

A positive relationship between the conditional mean and the conditional variance of aggregate stock returns (the risk-return tradeoff) is a central empirical implication of equilibrium asset pricing theory. Rational risk-averse investors require higher compensation in equilibrium for holding stocks during riskier periods, characterized by higher market volatility (Merton, 1973, 1980). Our paper adds to this explanation by showing that the market equity premium reflects greater compensation for holding stocks during more volatile periods *and* periods with high ambiguity aversion or pessimism. In contrast, this premium is weakened in optimistic periods, where more investors are motivated by lottery-like payoffs and positive skewness. We use ambiguity attitudes to formalize market optimism and pessimism and show theoretically and empirically that the interaction of market volatility and optimistic ambiguity attitudes restores a positive, stable risk-return tradeoff, and boosts return predictability. In contrast, the traditional risk-return tradeoff with only market volatility is insignificant and the coefficient is unstable across time.

There is a large and growing literature on the risk-return tradeoff. Ghysels et al. (2005) comment that "This risk-return trade-off is so fundamental in financial economics that it could be described as the 'first fundamental law of finance.' Unfortunately, the trade-off has been hard to find in the data. Previous estimates of the relation between risk and return often have been insignificant and sometimes even negative." Recent work continues to find the absence of a risk-return relation for the aggregate stock market (Moreira and Muir, 2017; DeMiguel et al., 2021; Barroso and Maio, 2023) and theoretical asset pricing models have struggled to offer an explanation.

In this paper, we build on the literature related to Knightian uncertainty and ambiguity in which probabilities of events are unknown and investors have varying degrees of optimism and pessimism (ambiguity attitudes) towards this uncertainty. Prior research has found that ambiguity and ambiguity aversion help explain the equity premium puzzle (*e.g.*, Chen and Epstein, 2002; Ju and Miao, 2012), the stock market non-participation puzzle (*e.g.*, Dow and da Costa Werlang, 1992; Easley and O'Hara, 2009; Dimmock et al., 2016), and the cross-section of expected stock returns (*e.g.*, Thimme and Völkert, 2015; Bali and Zhou, 2016). We provide a theoretical asset pricing model that shows market ambiguity attitude (the ambiguity attitude of the representative agent of the aggregate stock market) plays a critical role in explaining the risk-return tradeoff. To test if market ambiguity attitude affects the risk-return tradeoff, we first develop a methodology for measuring market ambiguity attitude directly from an asset pricing model.[1] The traditional risk-return tradeoff might suffer from an omitted variable bias, since volatility only captures one type of risk (Ghysels et al., 2005). The equity premium can be decomposed into the standard risk premium of the consumption CAPM, a speculative premium and an ambiguity premium. These additional premiums incorporate broader forms of risk as they depend on ambiguity, market optimism and pessimism, disaster risk, and positive skewness, linking four strands of the asset pricing literature. We show that market ambiguity attitude is related to the skewness of the risk-neutral distribution which is advocated by the CBOE as a measure of market tail risk. A survey of finance professionals finds that skewness is more important than volatility as a measure of risk (Holzmeister et al., 2020).

We consider a representative agent from the NEO-EU (non-extreme outcome expected utility) model of choice under ambiguity as in Chateauneuf et al. (2007) and Zimper (2012), which permits a full spectrum of ambiguity attitudes ranging from purely pessimistic to purely optimistic.[2] In contrast, the standard ambiguity models applied to market settings have difficulty reconciling both ambiguity-averse behavior and optimistic attitudes toward ambiguity (Gilboa and Schmeidler, 1989; Hansen and Sargent, 2001; Klibanoff et al., 2005; Kocher et al., 2018).

Chateauneuf et al. (2007) observe, "On an aggregate level, business cycles and stock market fluctuations have been attributed to 'irrational' optimism and pessimism. Economic theory, however, finds it difficult to see in such moods a major factor determining economic behavior." By demonstrating that market ambiguity attitude explains time variation in the risk-return tradeoff and that it predicts market crashes and recessions, our study identifies market ambiguity attitude as a missing state variable in traditional asset pricing theory that provides a new source for business cycles and stock market fluctuations.

[1]By "risk-return tradeoff" we refer to a very specific empirical relationship: the relationship between the conditional market excess return and the conditional market volatility. The presence of this tradeoff for the aggregate stock market (whether market volatility positively predicts excess returns) has been investigated and debated in many empirical studies. However, the empirical evidence has been mixed. French et al. (1987), Baillie and DeGennaro (1990), Campbell and Hentschel (1992), Ghysels et al. (2005), Lundblad (2007), Guo and Whitelaw (2006), Brandt and Wang (2007), and Pástor et al. (2008) find a positive risk-return tradeoff. In contrast, Campbell (1987), Nelson (1991), Whitelaw (1994), Brandt and Kang (2004), and Lettau and Ludvigson (2010) find a negative risk-return relation. As noted by Yu and Yuan (2011), "numerous studies over the past three decades find rather mixed empirical evidence of such a relation" (p.367).

[2]The NEO-EU model satisfies the axioms of both the α-maxmin multiple priors model (Gilboa and Schmeidler, 1989; Ghirardato et al., 2004), and Choquet expected utility theory (Schmeidler, 1989), two of the primary frameworks for modeling decisions under ambiguity in which objective probabilities of events are unknown.

In this paper, we focus on the new aspect of our approach which is the time series of market ambiguity attitude. The main contribution is to the literature on applications of ambiguity models to asset pricing and return predictability. First, we introduce a methodology for extracting market ambiguity attitude (i.e., optimism and pessimism of the representative agent) and aggregate market ambiguity. Second, we demonstrate theoretically and empirically that market ambiguity attitude generates time variation in the risk-return tradeoff. Third, results document that market ambiguity attitude predicts stock market crashes and recessions, identifying two sources of its predictive power for market returns.

A preview of our results shows that across our sample period, 1990 – 2022, market volatility fails to significantly positively predict the equity premium. However, when including the interaction between market ambiguity attitude and volatility, the coefficient on market volatility is positive and significant while the interaction term is negative and significant, as predicted by the theory in Section 2. The complementary predictability between volatility and market ambiguity attitude is strong, raising the in-sample R^2 by a factor of three or more for in-sample tests when the interaction term is included. The out-of-sample R-squared (R^2_{OS}) at the one-month forecast horizon increases from -0.16% to 4.06% when the interaction term is added to the regression.

As an additional out-of-sample analysis of the value of of the information contained in the market ambiguity attitude, we construct a real-time dynamic investment strategy based on our index. The strategy based volatility and market ambiguity attitude nearly doubles the Sharpe ratio of the historical average benchmark, increasing from 0.40 for the historical average to 0.78, while the certainty equivalent return rises from 4.07% to 13.17%. The investment strategy also generates a significant annualized Fama-French six-factor alpha of 7.06%. Goyal-Welch graphs further illustrate that the predictive performance of the forecast with market ambiguity attitude and market volatility consistently outperforms that of the historical average benchmark over the out-of-sample period while market volatility alone under-performs the historical average. As a secondary finding, market ambiguity attitude also forecasts NBER recessions even after controlling for sentiment and other recession predictors. We also show that ambiguity attitude restores the risk-return tradeoff over long horizons, whereas out-of-sample forecasts based on market volatility yield a negative R^2_{OS} at all horizons.

2 The NEO-EU CAPM

2.1 The Representative Agent

The α-maxmin multiple priors framework from Ghirardato et al. (2004), of which the NEO-EU model of Chateauneuf et al. (2007) is a notable special case, is a prominent axiomatic framework from decision theory in which aversion and affinity to ambiguity coexist. The $\alpha-$maxmin multiple priors framework and the NEO-EU model have been studied in laboratory experiments at the level of individual behavior (Baillon and Bleichrodt, 2015; Baillon et al., 2018; Dimmock et al., 2015; Kocher et al., 2018; König-Kersting et al., 2023) and in market experiments (Bossaerts et al., 2010). The α-maxmin model has also been applied to asset pricing theory (Chateauneuf et al., 2007; Zimper, 2012; Anthropelos and Schneider, 2022). However, it has not yet been applied to the risk-return tradeoff, and it has not been investigated empirically using stock market data.

Let $\mathcal{S}$ represent a compact set of possible future states of nature, $\mathcal{C}$ a set of consumption levels, and $\mathcal{F}$ a set of acts where an act, $f : \mathcal{S} \to \mathcal{C}$ assigns a consumption level to each state. One state $s \in \mathcal{S}$ will be realized but that true state is presently unknown. Subsets of $\mathcal{S}$ are referred to as *events*. Let Ω denote the set of all possible events. Let $\Delta(\mathcal{S})$ denote the set of all probability distributions on $\mathcal{S}$.

It is typical to write the α-maxmin value function with α as the weight on the worst-case expected utility. However, the original NEO-EU model formulation includes α as the weight on the best-case expected utility and in the present context that formulation is more intuitive to describe how α flattens the slope of the risk-return relation.

DEFINITION 1. *An α-maxmin agent has the following value function for an act f:*

$$V(f) \coloneqq \alpha \max_{\mathcal{P}\in M} E_{\mathcal{P}}\, u\big(C(s)\big) + (1-\alpha) \min_{\mathcal{P}\in M} E_{\mathcal{P}}\, u\big(C(s)\big), \tag{1}$$

where $M \subseteq \Delta(\mathcal{S})$ is a closed convex set of prior distributions that the agent deems plausible given the agent's information. In (1), α represents the agent's attitude (degree of optimism) toward ambiguity, and $E_{\mathcal{P}}\, u\big(C(s)\big)$ is the agent's expected utility with respect to prior distribution $\mathcal{P} \in M$.

In empirical applications, it is often useful to assume a parameterized set of prior distributions.

A common specification of M is the following (Chateauneuf et al., 2007; Dimmock et al., 2015):

$$M_\gamma = \{\mathcal{P} \in \Delta(\mathcal{S}) : \mathcal{P}(E) \geq (1-\gamma)\pi(E)\}, \tag{2}$$

for all $E \in \Omega$, where $\gamma \in [0,1]$. In (2), the agent has a reference prior distribution, π, and a degree of confidence in that reference prior of $1-\gamma$. As summarized in Dimmock et al. (2015), the set of priors M_γ implies the following restrictions on the probability distributions $\mathcal{P} \in M_\gamma$:

$$0 \leq (1-\gamma)\pi(E) \leq \mathcal{P}(E) \leq (1-\gamma)\pi(E) + \gamma \leq 1,$$

for all $E \in \Omega$. Dimmock et al. (2015) note that the set of priors M_γ "allows the probability $\mathcal{P}(E)$ to vary in an interval of length γ around the reference probability $\pi(E)$." In this model, γ is interpreted as the level of perceived ambiguity and the model reduces to the standard subjective expected utility model when the agent perceives no ambiguity (corresponding to $\gamma = 0$).

Chateauneuf et al. (2007) show that the α-maxmin model in (1) combined with the set of priors in (2) is equivalent to the NEO-EU representation of preferences in Equation (3) for which they provide an axiomatic foundation. A NEO-EU (non-extreme outcome expected utility) agent maximizes a weighted average of the expected utility of an uncertain prospect and the Hurwicz value of the prospect which takes a convex combination of the best and worst-case utilities.[3]

DEFINITION 2. *A NEO-EU agent has the following value function for an act f:*

$$V(f) = (1-\gamma)E_\pi u\big(C(s)\big) + \gamma\big(\alpha\, u(\overline{C}) + (1-\alpha)u(\underline{C})\big). \tag{3}$$

In (1), $V(f)$ is the valuation of act f for the NEO-EU agent, $E_\pi u\big(C(s)\big)$ is the agent's expected utility (EU) from consumption under act f with respect to her subjective probability distribution, π, while $u(\overline{C})$ and $u(\underline{C})$ are, respectively, the utility from the best-case and worst-case consumption levels across states under f. These preferences separate the agent's beliefs, ambiguity attitude, α, and perceived level of Knightian uncertainty, γ. The agent's ambiguity attitude can range from pure ambiguity aversion or pure pessimism ($\alpha = 0$) to pure ambiguity seeking or pure optimism ($\alpha = 1$). The agent's perceived level of ambiguity, γ, ranges from no ambiguity ($\gamma = 0$), in which case the

[3] We restrict our attention to acts that are simple functions as in Lemma 3.1 of Chateauneuf et al. (2007).

agent maximizes expected utility with respect to her subjective prior distribution, π, to complete uncertainty ($\gamma = 1$), where the agent places no confidence in her prior and relies on the Hurwicz criterion for robust decision making which is robust to all prior distributions over the same support. The NEO-EU model nests expected utility preferences ($\gamma = 0$), and the ϵ-contamination model of ambiguity aversion ($\gamma \in (0,1]$, $\alpha = 0$), two prominent theoretical benchmarks in the literature (Dow and da Costa Werlang, 1992).

The NEO-EU model accommodates aversion toward left-tail ambiguity and a preference for speculating on right-tail ambiguity. By overweighting the extreme outcomes, Chateauneuf et al. (2007) show that the NEO-EU model explains the behavior of a consumer who purchases both lottery tickets and insurance, which has been a challenge for EU since its inception (Friedman and Savage, 1948; Ebert and Karehnke, 2021). More generally, the NEO-EU model generates a preference for ambiguity over low-likelihood events and an aversion to ambiguity over high-likelihood events, consistent with the experiments in Baillon and Bleichrodt (2015) and Kocher et al. (2018). Since the focus of our empirical strategy is to capture the low-frequency movements in the risk-neutral probability of tail events, NEO-EU is a natural choice among ambiguity models for our application. In contrast this overreaction to both positive and negative tail events is not captured by popular ambiguity models that permit only uniform ambiguity attitudes, such as the smooth model of ambiguity aversion (Klibanoff et al., 2005), the maxmin multiple priors model, (Gilboa and Schmeidler, 1989), robust control preferences (Hansen and Sargent, 2001), and the ϵ-contamination model (Dow and da Costa Werlang, 1992).[4]

2.2 Equilibrium

Motivated by Chateauneuf et al. (2007) and Zimper (2012), we consider an asset pricing model with a *NEO-EU* representative agent. Our goal is not to develop a full-fledged dynamic general equilibrium model, but rather to develop a transparent model that highlights that the risk-return tradeoff is missing a role for market ambiguity attitude and that provides testable implications. As in Chateauneuf et al. (2007), we present our analysis in a simple two-period model in which the

[4]The NEO-EU model also has an alternative interpretation of based on probability weighting. Wakker (2010) notes that the probability weighting function embedded in (3), is among the most promising families of weighting functions in the literature and "the interpretation of its parameters is clearer and more convincing than with other families."

economy has one risky asset representing the aggregate stock market and a risk-free zero-coupon bond in zero net supply. The risky asset's price in period t is P_t, and its stochastic payoff in state s in period $t+1$ is $X_{t+1}(s)$. The risk-free bond's price in period t is P_t^b, and its payoff is one unit of consumption with certainty. We assume the agent has a standard CRRA (constant relative risk aversion) utility function. The agent's utility in period t is $u(C_t) := \frac{C_t^{1-\lambda}-1}{1-\lambda}$, where C_t is the current level of consumption and λ is the relative risk aversion parameter. We denote the time discount rate by $\delta \in (0,1)$. To simplify notation, in our subsequent analyses, for any variable θ, we define $\theta_{t+1} := \theta_{t+1}(s)$, and we denote the corresponding conditional expectation by $E_t\theta_{t+1} := E_{\pi,t}\theta_{t+1}(s)$. At time t, the agent chooses its level of consumption and investment to maximize:

$$\max_{\{C_t,B_t,S_t\}} u\big(C_t\big) + (1-\gamma_t)\delta E_t u\big(C_{t+1}\big) + \gamma_t\delta\big[\alpha_t\, u\big(\overline{C}_{t+1}\big) + (1-\alpha_t)u\big(\underline{C}_{t+1}\big)\big], \tag{4}$$

where $E_t u(C_{t+1})$ is the time t expected utility of consumption in period $t+1$, and $u(\overline{C}_{t+1})$ and $u(\underline{C}_{t+1})$ are utilities from the perceived best (optimistic) and worst (pessimistic) case consumption levels in period $t+1$. Note that the conditional expected utility, $E_t u(C_{t+1})$, and the conditional maximum and minimum consumption levels, $u(\overline{C}_{t+1})$ and $u(\underline{C}_{t+1})$ are known to the agent at time t. Moreover, γ_t and α_t represent the agent's perceived *ambiguity* and *ambiguity attitude* at time t. The budget constraints at time t and $t+1$ are $C_t + P_t^b B_t + P_t S_t = \Omega_t$, and $C_{t+1} = B_t + S_t X_{t+1} + \Omega_{t+1}$, where S_t and B_t are the agent's position in the risky and risk-free assets in time t, and Ω_t is the agent's endowment at time t.

The utility cost of each unit of (forgone) consumption at time t is $u'(C_t)$, and since one share of stock is worth P_t units of consumption, the utility cost of buying one unit of stock is $P_t\, u'(C_t)$. The payoff from one share of stock in $t+1$ is X_{t+1}, and thus, the expected utility gains from buying a share of stock is $(1-\gamma_t)\delta E_t u'(C_{t+1})X_{t+1} + \gamma_t\delta\big(\alpha_t u'(\overline{C}_{t+1})\overline{X}_{t+1} + (1-\alpha_t)u'(\underline{C}_{t+1})\underline{X}_{t+1}\big)$. Thus, the equilibrium price P_t adjusts to equate the current marginal utility cost to the discounted marginal utility gains, that is:

$$P_t\, u'(C_t) = (1-\gamma_t)\delta E_t u'(C_{t+1})X_{t+1} + \gamma_t\delta\big(\alpha_t u'(\overline{C}_{t+1})\overline{X}_{t+1} + (1-\alpha_t)u'(\underline{C}_{t+1})\underline{X}_{t+1}\big).$$

We assume the representative agent ranks the state of the world where both consumption and market payoff are the highest (lowest) as the best (worst) state in $t+1$. Dividing the above equation by $u'(C_t)$ and using the notation for the stochastic discount factor, $M_{t+1} := \delta\frac{u'(C_{t+1})}{u'(C_t)}$:

$$P_t = (1-\gamma_t)E_t M_{t+1} X_{t+1} + \gamma_t\big(\alpha_t \overline{M}_{t+1}\overline{X}_{t+1} + (1-\alpha_t)\underline{M}_{t+1}\underline{X}_{t+1}\big). \tag{5}$$

In (5), the price of a risky asset is a weighted average of the fundamental component, $E_t M_{t+1} X_{t+1}$, which is the asset's expected discounted payoff, that reflects the agent's information, and an ambiguity attitude component, $\alpha_t \overline{M}_{t+1}\overline{X}_{t+1} + (1-\alpha_t)\underline{M}_{t+1}\underline{X}_{t+1}$, that is a function of the agent's optimism and pessimism toward ambiguity. The relative strength of these two components depends on the agent's perceived level of ambiguity in the market, γ_t, with the agent relying less on its information at times of high ambiguity. The effects of market ambiguity attitude are amplified in times of high ambiguity.

Given that the return in state s is the payoff in state s, divided by the price, we have $R_{t+1} = X_{t+1}/P_t$ and hence, we express (5) as the following Euler equation:

$$(1-\gamma_t)E_t M_{t+1} R_{t+1} + \gamma_t\left(\alpha_t \overline{M}_{t+1}\overline{R}_{t+1} + (1-\alpha_t)\underline{M}_{t+1}\underline{R}_{t+1}\right) = 1. \tag{6}$$

Similarly, for the return of the risk-free bond, $R_{f,t} = 1/P_t^b$, we have

$$R_{f,t}\Big((1-\gamma_t)E_t M_{t+1} + \gamma_t\big(\alpha_t \overline{M}_{t+1} + (1-\alpha_t)\underline{M}_{t+1}\big)\Big) = 1. \tag{7}$$

2.3 The Equity Premium

Subtracting (7) from (6) and rearranging terms yields the equity premium:

$$E_t R_{t+1} - R_{f,t} = \underbrace{-\frac{Cov_t(M_{t+1}, R_{t+1})}{E_t\left[M_{t+1}\right]}}_{\text{Risk Premium}} + \underbrace{\frac{\left[(R_{f,t} - \overline{R}_{t+1})\overline{M}_{t+1}\right]\alpha_t\gamma_t}{E_t\left[M_{t+1}\right](1-\gamma_t)}}_{\text{Speculation Premium}} + \underbrace{\frac{\left[(R_{f,t} - \underline{R}_{t+1})\underline{M}_{t+1}\right](1-\alpha_t)\gamma_t}{E_t\left[M_{t+1}\right](1-\gamma_t)}}_{\text{Ambiguity Premium}}. \tag{8}$$

Equation (8) is a generalization of the classical Consumption CAPM formula that decomposes the equity premium into three terms. The first term is the well-known risk-premium term from the case with an expected utility representative agent. We refer to the second term as a *speculation premium*, and it is negative, reflecting that investors pay to hold stocks that are more exposed to market optimism (or a market boom). Ceteris paribus, the speculation premium becomes larger in magnitude with higher market optimism, α_t, market positive skewness, $\overline{R}_{t+1}$, or market ambiguity, γ_t. The third term is an *ambiguity premium* that ceteris paribus becomes larger in magnitude with higher market ambiguity aversion, $(1-\alpha_t)$, market disaster risk (lower $\underline{R}_{t+1}$), or market ambiguity, γ_t. Equation (8) includes a role for market optimism (α_t), ambiguity (γ_t), positive skewness ($\overline{R}_{t+1}$), and disaster risk ($\underline{R}_{t+1}$), thereby unifying these strands of the asset pricing literature. Since (8) is derived from a NEO-EU representative agent, we refer to (8) as the *NEO-EU CAPM.*

2.4 Best and Worst States

To operationalize the model, we parameterize the best and worst-case scenarios perceived by the agent. To do so, let us have the following structure on the joint distribution of returns and consumption growth

$$\begin{bmatrix} r_{t+1} \\ \Delta c_{t+1} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_t \\ g \end{bmatrix}, \begin{bmatrix} q_t^2 & q_t\sigma\eta \\ q_t\sigma\eta & \sigma^2 \end{bmatrix} \right), \tag{9}$$

where $r_{t+1} \coloneqq \log(R_{t+1})$ and $\Delta c_{t+1} \coloneqq \log(\frac{C_{t+1}}{C_t})$. In the model, prices are endogenous to equate both sides of (8). Still, since payoffs and endowments are exogenous, the model puts no restriction on the joint distribution of returns and consumption, and (9) is a standard structure in the asset pricing literature with ample empirical support.[5]

ASSUMPTION 1. *In state $s \in \mathcal{S}$ in period $t+1$, the agent's perceived log return and log consumption growth rate are $r_{t+1}(s) = \mu_t + \xi_s q_t$ and $\Delta c_{t+1}(s) = g + \xi_s \sigma$, respectively.*

Under Assumption 1, the perceived highest and lowest returns across states are then $\overline{r}_{t+1} = \mu_t + \overline{\xi} q_t$ and $\underline{r}_{t+1} = \mu_t - \underline{\xi} q_t$, where $\overline{\xi} \coloneqq \max_{s\in\mathcal{S}} \xi_s$, and $\underline{\xi} \coloneqq |\min_{s\in\mathcal{S}} \xi_s|$. Similarly, the perceived highest and lowest consumption growth rates are $\overline{\Delta c}_{t+1} = g + \overline{\xi}\sigma$ and $\underline{\Delta c}_{t+1} = g - \underline{\xi}\sigma$. Assumption 1 specifies the perceived log return and consumption growth rate to be within an interval of their expected means, and the interval size increases with conditional volatility. In our empirical work, we consider the simplest case in which the endpoints of the interval are symmetric around the mean, i.e., $\xi \coloneqq \overline{\xi} = \underline{\xi}$.

[5] GARCH models are special forms of (9) where q_t itself has additional structure. Further, a multivariate GARCH model with both log return and log consumption growth rejects a time-varying correlation coefficient η_t.

2.5 Approximations

This subsection shows how the NEO-EU representative agent and the best and worst-case state Assumption 1 yield intuitive approximations for the equity premium and the variance risk premium.

PROPOSITION 1. *(Market ambiguity attitude and the risk-return tradeoff) Under Assumption 1 with* $\overline{\xi} = \underline{\xi} = \xi$*, the equity premium for a NEO-EU agent with the CRRA utility is approximately*

$$E_t R_{t+1} - R_{f,t} \approx \big(1 - 2\alpha_t + \lambda\sigma\xi\big)\xi\gamma_t q_t. \tag{10}$$

Proof. See the Appendix. □

The approximation in (10) is accurate for reasonable values of the risk aversion parameter, for example, $\lambda \in [0, 2]$.[6] The evidence for the accuracy for the approximation is presented following the proof in the appendix. Unsurprisingly, the contribution of the covariance term in (8) to the equity premium is negligible. However, a novel feature of the theory is that the risk aversion parameter λ affects the equity premium by amplifying the effect of ambiguity and volatility, although the contribution is still small. In the special case where $\lambda = 0$ (risk-neutrality), the equity premium approximation in (10) is exact and takes the following simple form:

$$E_t R_{t+1} - R_{f,t} = \big(1 - 2\alpha_t\big)\xi\gamma_t q_t. \tag{11}$$

The equity premium approximation in (11) most clearly distills the intuition for how ambiguity attitudes affect the market risk-return tradeoff: The conditional equity premium is increasing in market volatility, q_t, but the slope of this relationship flattens as market ambiguity attitude becomes more optimistic (as α_t increases).

The variance risk premium (VRP) is the difference between the risk-neutral and physical market variance, and it is commonly interpreted as a measure of economic uncertainty (Zhou, 2018; Bali and Zhou, 2016). Formally, the VRP is defined as $VRP_t := \mathrm{Var}_t^Q R_{t+1} - \mathrm{Var}_t R_{t+1}$ (Zhou, 2018), where Var_t^Q is the conditional variance under the risk-neutral measure. The following proposition presents a powerful formula to approximate the VRP for the NEO-EU agent.

[6] The seminal paper by Holt and Laury (2002) finds that the CRRA parameter estimated for their participants in lab experiments is typically between 0 and 1. They classify a value greater than 1.37 as "stay in bed." In investment applications, Ferreira and Santa-Clara (2011) and Jondeau et al. (2019) assume a risk aversion parameter of 2.

PROPOSITION 2. *Under Assumption 1 with $\overline{\xi} = \underline{\xi} = \xi$, the variance risk premium for a NEO-EU agent with the CRRA utility is approximately*

$$VRP_t \approx \gamma_t q_t^2(\xi^2 - 1). \tag{12}$$

Proof. See the Appendix. □

The evidence for the accuracy of approximation (12) is presented following the proof in the appendix. In Proposition 2, the VRP approximation is independent of the market ambiguity attitude, α_t. The propositions 1 and 2 thus provide a partial separation between ambiguity and ambiguity attitude. Further, Proposition 2 provides a theoretical link between a market-based measure of Knightian uncertainty (VRP) and a behavioral measure of Knightian uncertainty, γ_t, while γ_t (scaled by the constant $\xi^2 - 1$) serves as a wedge between VRP_t and q_t^2.

2.6 Taking the Model to Data

In this subsection, we use the model expressions from the previous subsections to extract a measure of ambiguity attitude α_t. As our main specification we use the case in which $\lambda = 0$ (risk-neutrality), corresponding to the representation in (11) which clearly illustrates the intuition linking market ambiguity attitude to the risk-return tradeoff. Theoretically, including a CRRA parameter $\lambda > 0$ has virtually no effect on our results which focus on the time variation in the equity premium and market crashes. We confirm this empirically in the Internet Appendix where we outline the construction of α_t for $\lambda > 0$ and show that the α_t series using the case of log utility ($\lambda = 1$) yields virtually identical empirical results in-sample and out-of-sample.

To construct α_t in our benchmark case with $\lambda = 0$, we first estimate q_t from a GARCH model and use it to construct a measure of γ_t. Then, α_t can be estimated using a Markov switching model. The details are provided below.[7] First, we measure q_t via a simple GARCH(1,1) model for the log

[7] Empirically, our estimated measure of γ_t has a first-order autocorrelation of 0.46, while α_t has a first-order autocorrelation of 0.97. The high persistence of α_t is consistent with the high persistent of market optimism that is noted in prior work (Schmeling, 2007). Theoretically, the low persistence of market ambiguity, γ_t, arises naturally if ambiguity changes (beliefs are updated and the ambiguity is resolved) in response to streams of new information each period. In contrast, ambiguity attitudes are a trait that one might expect to be largely stable over time and which depend on investors' education, age, or other demographic factors, but which are subject to slow-moving fluctuations (Grevenbrock et al., 2020) that may generate fluctuations in the optimism of the representative agent of the aggregate market. For instance, regarding the dot-com bubble which is the period with the highest level of α_t across our full sample period, Greenwood and Nagel (2009) find that "around the peak of the technology bubble, mutual funds run

returns, where pd_t is the price-dividend ratio for the S&P 500 index and is used to construct a measure of the expected equity premium:[8]

$$\log(R_{t+1}) = \theta_0 + \theta_1 pd_t + u_{t+1}, \tag{13}$$

$$u_{t+1} = q_{t+1} z_{t+1}, \quad \text{with} \quad z_{t+1} \sim \mathcal{N}(0,1) \tag{14}$$

$$q_{t+1}^2 = \omega_0 + \lambda_1 u_t^2 + \beta_1 q_t^2. \tag{15}$$

In this specification, the log market return is linear in the price-dividend ratio (pd_t), and the error is assumed to follow a GARCH(1,1) process.[9] Lemma 1 in Internet Appendix D shows that the log return is approximately linear in the price-dividend ratio when we replace the payoff with dividends. The estimation not only provides us with an estimate of the conditional market volatility $\hat{q}_t$, but also yields a conditional market equity premium based on the conditional expected return and the market risk free rate $E_t R_{t+1} - R_{f,t} \approx \exp(\hat{\theta_0} + \hat{\theta_1} pd_t) - R_{f,t}$.[10] For notational convenience, we denote the conditional equity premium by $\text{EP}_t \coloneqq E_t R_{t+1} - R_{f,t}$.

Having established a theoretical link between market ambiguity attitude and the risk-return tradeoff, our next objective is to provide an estimate of the time series of α_t using the structural equations from the model under Assumption 1 and linear utility.

Second, following Zhou (2018) and Bekaert and Hoerova (2014), we use the square of the VIX index as a proxy for the risk-neutral variance, $\text{Var}_t^Q R_{t+1}$. Then using formula (12) we find γ_t to be:

$$\hat{\gamma}_t \approx \frac{1}{\xi^2 - 1}\Big(\frac{\text{VIX}_t^2}{\hat{q}_t^2} - 1\Big). \tag{16}$$

Finally, we use the relationship, $\text{EP}_t = \xi(1 - 2\alpha_t) q_t \gamma_t$ from Equation (10) to estimate α_t. In line with the intuition that α_t has persistent dynamics, we let α_t follow a Markov-switching structure with two states. Ang and Timmermann (2012) motivate regime switching models since they match "the tendency of financial markets to often change their behavior abruptly and the phenomenon that the new behavior of financial variables often persists for several periods after such a change."

by younger managers are more heavily invested in technology stocks, relative to their style benchmarks, than their older colleagues."

[8] The dividend price ratio is also used as a proxy for the equity premium empirically in Pástor and Veronesi (2020) and is closely related to the equity premium in traditional macro-finance models.

[9] The results of the GARCH regressions (13) - (15) are presented in Table 12 in Internet Appendix B.

[10] Allowing for the second order (Jensen) term virtually makes no difference in the final estimates.

In the case of stock return predictability, a two-state regime-switching model can correspond to bull and bear markets or expansions and recessions (Rapach and Zhou, 2013). Note that the relationship implies $\xi(1-2\alpha_t) = \dfrac{\mathrm{EP}_t}{q_t\gamma_t}$. Thus, if α_t follows a Markov-switching model, so does the ratio $\dfrac{\mathrm{EP}_t}{q_t\gamma_t}$. To estimate α_t, we estimate the following Markov-switching dynamic regression model:

$$\frac{\hat{\mathrm{EP}}_t}{\hat{q}_t\hat{\gamma}_t} = \mu_{m_t} + \epsilon_t, \tag{17}$$

where ϵ_t is a white noise and μ_{m_t} switches between two regimes according to a probability matrix.

The quantity $\dfrac{\mathrm{EP}_t}{q_t\gamma_t}$ is a measure like a conditional Sharpe ratio but which includes a role for market ambiguity, γ_t. In the Markov-Switching model there are two regimes: (i) a bear market regime with relatively low prices and high expected future returns per unit of risk, and (ii) a bull market regime with relatively high prices and low expected future returns per unit of risk. Market optimism, α_t, is then increasing in the probability of the bull market regime.

The dynamically estimated model gives us a predicted value of $\hat{\mu}_{m_t}$ using the information up to and including time t to avoid look-ahead bias. We then find our estimate of α_t according to:

$$\hat{\alpha}_t = \frac{1}{2}\Big(1 - \frac{\hat{\mu}_{m_t}}{\xi}\Big). \tag{18}$$

As ambiguity by itself has received much attention in prior literature, we focus on the time series of market ambiguity attitude. This focus also reflects the motivation of the paper which is to investigate if market ambiguity attitude restores the risk-return tradeoff which is predicted by the theory studied here. Assuming γ_t has little or no predictive content due to its low autocorrelation, q_t and α_t contain all of the information about the conditional equity premium in Equation (10).

2.7 The Level of Market Ambiguity and Ambiguity Attitude

We calibrate ξ, which parameterizes the number of standard deviations that form the interval around the mean return, to 4.77 since it implies unconditional best-case return and worst-case returns that are roughly consistent with common definitions of a bull market and a bear market in the media and on Wall Street (Kurov, 2010) and since it is also consistent with the 20% threshold used by Martin (2017) for a market crash. The specification does not rely on information in the out-of-sample period. Over the training sample period, the monthly mean market volatility, q, is 4.05% and the monthly mean expected log market return from the GARCH model is 0.74%. Computing the maximum return under Assumption 1 with these values yields $\overline{R} = 0.0074 + 4.77(0.0405) \approx 0.20$. This is consistent with the threshold for a bull market (a return of 20% from a market's recent low). The corresponding worst-case return is approximately -0.19, and similar to the threshold for a bear market (a return of -20% from a market's recent high) as noted by Kurov (2010). Since ξ is constant, ξ affects the level but not the time variation of α_t. Consequently, our results are robust to different values of ξ. Of particular note, our main metrics for forecast evaluation (the in-sample and out-of-sample R^2 and the difference in cumulative sum of squared forecast errors presented in Section 3) are identical for all $\xi \in [2, 4.79]$.[11] For $\xi = 4.77$, the corresponding mean value of α_t is 0.27 (with a standard deviation of 0.16), reflecting a moderate level of ambiguity aversion, and $\gamma_t = 0.05$ (with a standard deviation of 0.06), indicating that the representative agent of the aggregate market is, on average, close to the expected utility benchmark ($\gamma_t = 0$).

2.8 Market Ambiguity Attitude and Market Risk-Neutral Skewness

This subsection motivates and illustrates what α_t captures empirically. Intuitively, a smaller α_t, consistent with a more pessimistic NEO-EU representative agent, leads to greater negative skewness of the risk-neutral probability density. That is, one might expect the skewness of the risk-neutral density to increase in α_t. We also expect two other stock market variables, the risk-free rate and the market price-dividend ratio to be increasing in α_t as these predictions follow theoretically from Equation (7) and Lemma 1, respectively. To test these predictions, we correlate α_t with market risk-neutral skewness (RNS), the log risk-free rate (r_f), and the price-dividend ratio (pd). We also include the two other stock market variables (q and VIX) that, along with pd, are used in the construction of α_t. The correlations are shown in Figure 1. We find that α_t is positively and significantly related to r_f, pd, and RNS, supporting the theoretical predictions and intuition.

We plot α_t and RNS to visualize the relationship, and we conduct Granger causality tests to infer potential dependencies between α_t and RNS.[12] We find a positive and significant correlation

[11] A positive feature of a specification with $\xi \leq 4.79$ is that $\alpha_t \in (0, 1)$ for all periods in our sample spanning more than 30 years of monthly data. Under specifications with $\xi \geq 4.80$, the estimated α becomes negative in some periods. Truncating α_t at zero in those periods will slightly affect the R^2. We further clarify that ξ should be large enough to leave the distribution virtually unchanged, which is why we view 2 as a natural lower bound for ξ.

[12] Market risk-neutral skewness is measured from the SKEW index of the CBOE. The SKEW index was introduced

Table 1. Granger causality tests between α and Market Risk-Neutral Skewness

	BIC	AIC
$\alpha \nrightarrow$ RNS	0.010**	0.035**
RNS $\nrightarrow \alpha$	0.848	0.807

Notes: The Granger causality tests are conducted for both the optimal lag length (one period) under the Bayesian Information Criterion (BIC) and for the optimal lag length (two periods) under the Akaike Information Criterion (AIC). The p-values of the tests are reported. ** denotes the 5% level of statistical significance. The tests are for the out-of-sample period (2006:07 through 2022:12).

of 0.40 between α_t and RNS across the out-of-sample period. When graphing this relationship, shown in Figure 1, it is apparent that α_t looks like a smooth version of RNS. These observations indicate that market ambiguity attitude, α_t, contains information about low-frequency movements (and hence the more predictable variation) in RNS. We next conduct Granger causality tests using the optimal lag lengths according to the Bayesian Information Criterion (one period) and according to the Akaike Information Criterion (two periods). As shown in Table 1, α_t significantly Granger causes RNS, whereas RNS does not Granger cause α_t, implying that α_t predicts RNS.

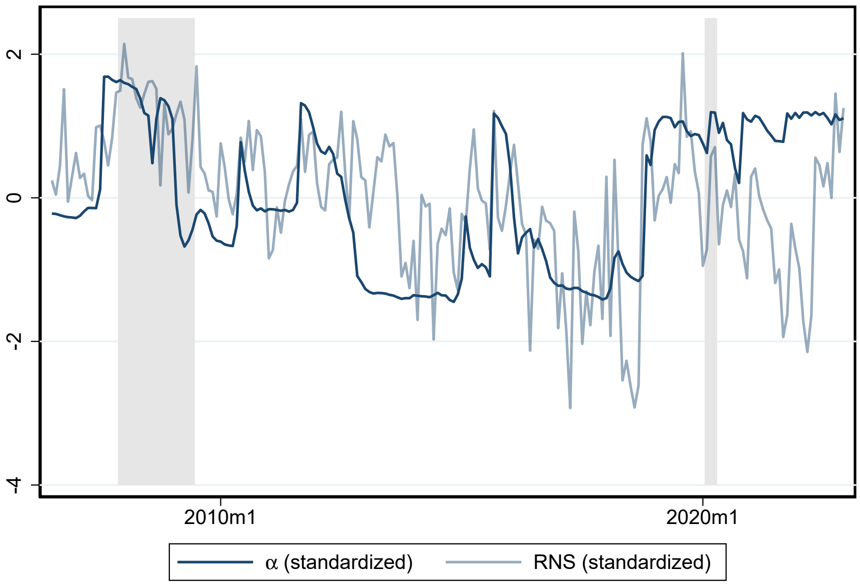
3 Market Ambiguity Attitude and the Risk-Return Tradeoff

3.1 Data Sources

We obtain data on the market excess return and risk-free rate from Kenneth French's data library which we use to construct the log equity premium. We obtain data on VIX, from the website of the CBOE. We obtain data on the market price-dividend ratio (pd) from Robert Shiller's website. These variables are used in the construction of our measure of market ambiguity attitude as outlined in Section 2.6. The sources of all data series used in our analysis are provided in Appendix A of the Internet Appendix. Summary statistics of the log equity premium, pd, and VIX, along with q_t, α_t, and $\alpha_t q_t$ (the primary variables used in our analysis) are provided in Table 2. Our control variables noted in the following sections are provided in Internet Appendix A.

as an "indicator that measures perceived tail risk" and the CBOE notes that it is intended to be complementary to the VIX index (CBOE, 2011). Formally, RNS $= E[(\frac{R-\mu}{\sigma})^3]$, where R is the 30-day log-return on the S&P 500, μ and σ are respectively the mean and standard deviation of R, $x := (\frac{R-\mu}{\sigma})^3$ and RNS $=$ E$[x]$. RNS is obtained from the SKEW index by the relation RNS $= \frac{100-SKEW}{10}$. The series is converted from the daily frequency to the monthly frequency using the last observation in each month as the RNS value for that month.

Figure 1. NEO-EU Optimism (α) and Market Risk-Neutral Skewness (RNS)



Correlations between α and Aggregate Stock Market Variables

	α	q	VIX	pd	r_f	RNS
α	1.00					
q	0.44***	1.00				
VIX	0.48***	0.66***	1.00			
pd	0.22***	-0.31***	-0.29***	1.00		
r_f	0.23***	-0.29***	-0.16**	0.12*	1.00	
RNS	0.40***	0.32***	0.36***	-0.37***	0.22***	1.00

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The figure displays the time series of the market risk-neutral skewness (RNS) from the Chicago Board of Options Exchange, and the recursively updated market ambiguity attitude series (α). The figure spans the out-of-sample period for α (the second half of the sample, 2006:07 through 2022:12). Over this period, the correlation between the two series is 0.40. For the purpose of comparison, both series have been standardized to have a mean of zero and a standard deviation of one over this period. Shaded areas show NBER recession periods. The table displays the correlations between six variables related to the aggregate stock market over the out-of-sample period: (i) the index of market ambiguity attitude, α_t; (ii) the conditional stock market volatility, q; (iii) VIX; (iv) the price-dividend ratio of the S&P 500 (pd); (v) the log risk-free rate, r_f; and (vi) RNS.

Table 2. Summary Statistics of Primary Variables

Variable	Mean	Std Dev.	Skewness	Kurtosis	Min	Median	Max
r_t^e	0.57	4.49	-0.76	4.45	-18.89	1.18	12.80
pd_t	52.96	13.91	0.41	2.97	25.75	51.76	90.21
VIX_t	5.71	2.20	1.64	7.10	2.75	5.22	17.29
q_t	4.21	1.46	1.25	5.05	2.18	3.95	10.18
α_t	0.27	0.16	0.28	1.86	0.00	0.20	0.52
$\alpha_t q_t$	1.24	0.97	0.76	2.59	0.01	0.82	4.63

Notes: This table reports the monthly summary statistics of the primary variables used in our analysis: The realized log market excess return, r_t^e (in percent); VIX_t (in percent), the price-dividend ratio (pd_t) of the S&P 500 index, the conditional market volatility, q_t (in percent), estimated from the GARCH(1,1) model in Section 2.6; market ambiguity attitude, α_t; and the product $\alpha_t q_t$. Values are rounded to the nearest two decimal places.

3.2 Testing the Risk-Return Tradeoff

Under Proposition 1, there is a positive relationship between the conditional market volatility, q_t, and the expected equity premium, but the slope of this relationship flattens as market optimism, α_t, increases. That is, the expected equity premium is increasing in q_t but decreasing in $\alpha_t q_t$. Our first analysis is motivated by three basic questions: First, does market ambiguity attitude moderate the risk-return tradeoff as predicted by Proposition 1? Second, if so, is the predictive power of market volatility, q_t, and the interaction between market ambiguity attitude and market volatility, $\alpha_t q_t$ subsumed by standard equity premium predictors? Third, what is the incremental increase in predictive power generated by including $\alpha_t q_t$ in the predictive regressions? To probe these questions, we consider 25 equity premium predictors consisting of the 14 predictors in Welch and Goyal (2008) available at the monthly frequency and the 11 newer predictors used by Cederburg et al. (2023) for which data is available beginning in 1990.[13] For q_t and α_t, our data begins in 1990 (the first year available for VIX which is needed for the construction of α_t).

Table 3 reports predictive regressions following Equation (19). In the full specification, the dependent variable is the (realized, cumulative) log equity premium, denoted $r^e_{[t+1,t+h]}$, where $h \in \{1, 3\}$, corresponding to the one-month log equity premium in period t and the cumulative three-month log equity premium, respectively. The lagged predictors include market volatility (q_t), the product of ambiguity attitude and volatility ($\alpha_t q_t$), and a set of k alternative predictors. We

[13] This criterion enables us to include predictors that span the period 1990 - 2021, and omits only the left-tail jump variation (LJV) predictor from Cederburg et al. (2023) for which available data begins in 1996. Including that variable does not affect the results.

consider the case with lagged volatility as the only regressor, as well as cases with $\alpha_t q_t$ and controls.

$$r^e_{[t+1,t+h]} = \beta_0 + \beta_1 q_t + \beta_2 \alpha_t q_t + \Sigma^k_{i=3} \beta_i x_{i,t} + \epsilon_{[t+1,t+h]}. \quad (19)$$

In Equation (19), k denotes the total number of predictor variables in the regression. Odd-numbered regressions in Table 3 do not include $\alpha_t q_t$, while this term is included in even-numbered regressions. Regressions summarized in columns (1), (2), (9), and (10) show our baseline results without controls. All data is updated from the original studies to span from 1990:02 - 2021:12.

In Table 3, for ease of interpreting the estimated coefficients, the predictors q_t and $\alpha_t q_t$ are divided by their full-sample standard deviation. Regression specification (2) in the top panel of Table 3 which includes both q_t and $\alpha_t q_t$ reveals that a one standard deviation increase in the conditional market volatility leads to an increase in the future realized log equity premium of 1.35% per month. In contrast, a one standard deviation increase in the product of market volatility and market optimism, $\alpha_t q_t$, leads to a decrease in the future equity premium of -1.36% per month. Both coefficients have t-statistics above three, and are economically large.

Table 3 answers our three questions. Regarding the first question, the table shows in Column (1) of Panels A and B that the relationship between r^e_{t+1} and lagged market volatility is not significant at the monthly or quarterly horizon. These regressions confirm the findings of Campbell (1987) and Barroso and Maio (2023) that the risk-return relationship is neither strong nor robust in the data. The regressions reveal that the absence of a risk-return tradeoff for the aggregate stock market continues to be a puzzle even when using more recent data than prior studies. In contrast, adding the interaction between market ambiguity attitude and market volatility to the regression yields a significant positive relation between r^e_{t+1} and lagged market volatility, q_t, and a significant negative relationship between r^e_{t+1} and lagged $\alpha_t q_t$ as noted in the preceding paragraph. These findings support the theoretical prediction that market ambiguity attitude restores the risk-return tradeoff.

Regarding our second question, Table 3 shows that neither the Welch and Goyal (2008) or Cederburg et al. (2023) predictors subsume the risk-return tradeoff results. In all eight specifications where q_t and $\alpha_t q_t$ are both included in the regressions, the coefficient on q_t is positive and significant while the coefficient on $\alpha_t q_t$ is negative and significant, even in the presence of 25 standard equity premium predictors as controls. In the absence of $\alpha_t q_t$, the coefficient on q_t is significant in the

Table 3. Market Ambiguity Attitude and the Risk-Return Tradeoff controlling for 25 Predictors

Monthly Forecast Horizon

Panel A	(1) r^e_{t+1}	(2) r^e_{t+1}	(3) r^e_{t+1}	(4) r^e_{t+1}	(5) r^e_{t+1}	(6) r^e_{t+1}	(7) r^e_{t+1}	(8) r^e_{t+1}
q_t	0.30	1.35***	1.49***	2.73***	-0.10	1.71***	1.10**	2.86***
	(1.18)	(5.44)	(4.07)	(4.45)	(-0.28)	(2.98)	(2.34)	(3.56)
$\alpha_t q_t$		-1.36***		-1.90***		-2.10***		-2.34***
		(-3.87)		(-2.70)		(-4.40)		(-3.18)
GW_t	NO	NO	YES	YES	NO	NO	YES	YES
CJO_t	NO	NO	NO	NO	YES	YES	YES	YES
k	1	2	15	16	12	13	26	27
adj. R^2	0.002	0.037	0.050	0.078	0.041	0.083	0.080	0.113

Quarterly Forecast Horizon

Panel B	(9) $r^e_{[t+1,t+3]}$	(10) $r^e_{[t+1,t+3]}$	(11) $r^e_{[t+1,t+3]}$	(12) $r^e_{[t+1,t+3]}$	(13) $r^e_{[t+1,t+3]}$	(14) $r^e_{[t+1,t+3]}$	(15) $r^e_{[t+1,t+3]}$	(16) $r^e_{[t+1,t+3]}$
q_t	0.92	3.89***	3.70***	6.68***	-1.29	3.49***	1.92**	5.86***
	(1.38)	(5.63)	(4.92)	(6.63)	(-1.46)	(3.66)	(2.08)	(4.57)
$\alpha_t q_t$		-3.86***		-4.53***		-5.55***		-5.23***
		(-4.07)		(-3.50)		(-6.71)		(-4.66)
GW_t	NO	NO	YES	YES	NO	NO	YES	YES
CJO_t	NO	NO	NO	NO	YES	YES	YES	YES
k	1	2	15	16	12	13	26	27
adj. R^2	0.011	0.108	0.188	0.241	0.158	0.256	0.291	0.346

Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium (in percent) against market volatility, q, and the set of 14 monthly equity premium predictors in Welch and Goyal (2008), the set of 11 newer equity premium predictors used by Cederburg et al. (2023) for which data are available beginning in January, 1990, and both sets of predictors. Even-numbered regressions also include αq. In the regression specifications in Panel A, the dependent variable is the one-month log equity premium, r^e_{t+1}, and all predictor variables are lagged by one month (monthly forecast horizon). In the regression specifications in Panel B, the dependent variable is the cumulative three-month log equity premium, $r^e_{[t+1,t+3]}$, and all predictors are lagged three months (quarterly forecast horizon). The GW row indicates whether the 14 monthly Welch and Goyal (2008) predictors are included as controls. The CJO row indicates whether the 11 Cederburg et al. (2023) predictors available starting in January, 1990, are included as controls. k denotes the number of predictor variables in the regression including q_t and the control variables, and $\alpha_t q_t$ where applicable. For ease of interpreting the coefficients, q_t and $\alpha_t q_t$ are divided by their (full sample) standard deviation. The sample period spans monthly data from 1990:01 through 2021:12.

presence of the Welch and Goyal (2008) predictors. Hence, the large set of predictors also affects the risk-return tradeoff. This result is consistent with the Welch and Goyal (2008) predictors representing time-varying macroeconomic risk, which is complementary to q_t and $\alpha_t q_t$.

Regarding the third question, Table 3 shows that including $\alpha_t q_t$ in the kitchen sink regressions substantially improves the predictive power. For instance, the adjusted R^2 for regression specification (5) in the table which includes the 11 recent predictors in Cederburg et al. (2023) is 4.1%. Adding $\alpha_t q_t$ to that regression roughly doubles the adjusted R^2 to 8.3%. The adjusted R^2 in regression specification (7) which includes q_t in addition to 25 established equity premium predictors is 8%. Adding $\alpha_t q_t$ to that regression increases the adjusted R^2 by over 3 percentage points to 11.3%. That α_t substantially enhances return predictability and the significance of market volatility in the presence of 25 standard equity premium predictors further supports our conclusion that α_t is a missing state variable that restores the risk-return tradeoff.

3.3 Sentiment, Ambiguity, Disagreement, and the Risk-Return Tradeoff

We next test the robustness of our findings to the inclusion of other variables that are potentially related to the risk-return tradeoff. Do these additional variables subsume the predictability of market ambiguity attitude? Yu and Yuan (2011) find that market sentiment, proxied by the Baker and Wurgler (2006) market sentiment index, affects the risk-return tradeoff. Since the Baker and Wurgler (2006) index and α_t are both measures of aggregate market optimism, we anticipate they will be positively related. Indeed we find they have a significant 0.39 correlation. We test here if the sentiment index subsumes the explanatory power of market ambiguity attitude. To investigate this, we run versions of regression (20) which includes q_t, $\alpha_t q_t$, and a control variable, x_t:

$$r^e_{[t+1,t+h]} = \beta_0 + \beta_1 q_t + \beta_2 \alpha_t q_t + \beta_3 x_t + \epsilon_{[t+1,t+h]}. \tag{20}$$

$x_t \in \{sentiment_t, disaster\ probabilities_t, ambiguity_t, PLS\ disagreement_t, analyst\ disagreement_t, average\ skewness_t, risk\ aversion_t\}$ and $h \in \{1, 3\}$. In addition to the Baker and Wurgler (2006) index (*sentiment*$_t$), we use other control variables, including the time-varying U.S. disaster probabilities of Barro and Liao (2021) (*disaster probabilities*$_t$), the ambiguity measure of Brenner and Izhakian (2018) (*ambiguity*$_t$), the partial least squares (PLS) disagreement measure of Huang et al. (2021) (*PLS disagreement*$_t$), the analyst disagreement measure of Yu (2011) (*analyst disagreement*$_t$), the value-weighted average skewness of Jondeau et al. (2019) (*average skewness*$_t$), and the time-varying risk aversion of Bekaert et al. (2022) (*risk aversion*$_t$). We also conduct a kitchen sink regression including all controls.[14]

Table 4 shows that the coefficient on volatility remains positive and significant and the coefficient on the interaction term $\alpha_t q_t$ remains negative and significant, in the presence of all controls. The kitchen sink regressions reveal that a one-standard deviation increase in q_t predicts a 1.32% increase in monthly returns and a 3.57% increase in quarterly returns, while a one standard deviation increase in $\alpha_t q_t$ predicts a 1.59% decrease in monthly returns and a 4.76% decrease in quarterly returns. Adding $\alpha_t q_t$ to the kitchen sink regressions increases the adjusted R^2 by 2.1 percentage points at the one-month horizon, and by 6.5 percentage points at the quarterly horizon. In contrast, the control variables are generally insignificant and their predictive power is not robust across both monthly and quarterly horizons with the exception of the PLS disagreement index.

3.4 Out-of-Sample Regressions

Following Welch and Goyal (2008), it is increasingly common to test if evidence of return predictability from in-sample regressions also holds out-of-sample. Consequently, we investigate the risk-return tradeoff and the predicted moderating effect of α in out-of-sample regressions. We use three standard metrics to evaluate out-of-sample (OOS) predictability: (1) the R^2_{OS} statistic of Campbell and Thompson (2008); (2) the MSPE-adjusted statistic of Clark and West (2007) which we use to measure the statistical significance of the predictability; and (3) the difference in cumulative sum of squared errors between the historical average equity premium forecast and the forecast based on predictor variables (Welch and Goyal, 2008). The R^2_{OS} statistic is defined in (21):

$$R^2_{OS} = 1 - \frac{\sum_{t=1}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=1}^{T}(r_t - \overline{r}_t)^2}, \tag{21}$$

where r_t is the realized log equity premium, $\hat{r}_t$ is the forecast from the predictive regression using information through period t, and $\overline{r}_t$ is the mean historical equity premium through period t.

[14] The measure in Brennan et al. (2004) is an index of ambiguity whereas α_t in the present paper is a measure of ambiguity attitude. In contrast to our approach, the ambiguity attitude in Brennan et al. (2004) is not time-varying.

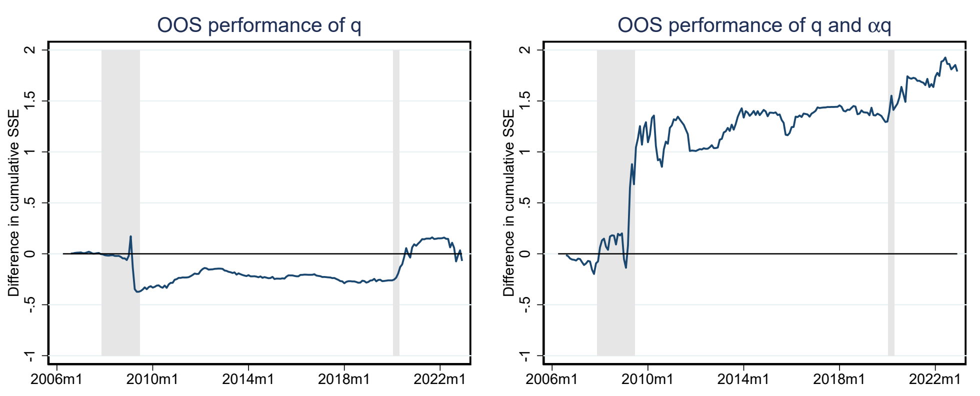
Table 4. Market Ambiguity Attitude and the Risk-Return Tradeoff with additional controls

Monthly Horizon Predictor	(1) r^e_{t+1}	(2) r^e_{t+1}	(3) r^e_{t+1}	(4) r^e_{t+1}	(5) r^e_{t+1}	(6) r^e_{t+1}	(7) r^e_{t+1}	(8) r^e_{t+1}
q_t	1.29***	1.21***	1.41***	0.92***	1.41***	1.37***	1.30***	1.32**
	(4.63)	(3.10)	(4.78)	(4.01)	(5.35)	(5.28)	(3.10)	(2.46)
$\alpha_t q_t$	-1.23***	-1.47***	-1.50***	-0.79*	-1.30***	-1.29***	-1.37***	-1.59**
	(-2.75)	(-4.66)	(-3.98)	(-1.91)	(-3.68)	(-3.55)	(-4.16)	(-2.38)
$sentiment_t$	-0.15							0.04
	(-0.53)							(0.15)
$disaster\ probabilities_t$		0.39						1.75*
		(0.53)						(1.70)
$ambiguity_t$			-0.38					-0.15
			(-1.38)					(-0.40)
$PLS\ disagreement_t$				-0.66***				-0.44
				(-3.05)				(-1.60)
$analyst\ disagreement_t$					-0.11			-0.79***
					(-0.52)			(-2.91)
$average\ skewness_t$						-0.11		-0.21
						(-0.36)		(-0.73)
$risk\ aversion_t$							0.09	-1.38
							(0.14)	(-1.27)
N	389	319	344	347	383	383	394	292
adj. R^2	0.037	0.043	0.046	0.052	0.037	0.037	0.035	0.070
Δ adj. R^2	0.021	0.042	0.044	0.008	0.034	0.033	0.036	0.021
Quarterly Horizon Predictor	(9) $r^e_{[t+1,t+3]}$	(10) $r^e_{[t+1,t+3]}$	(11) $r^e_{[t+1,t+3]}$	(12) $r^e_{[t+1,t+3]}$	(13) $r^e_{[t+1,t+3]}$	(14) $r^e_{[t+1,t+3]}$	(15) $r^e_{[t+1,t+3]}$	(16) $r^e_{[t+1,t+3]}$
q_t	3.47***	2.83***	3.88***	2.56***	4.01***	3.76***	3.64***	3.57***
	(4.50)	(2.71)	(4.80)	(4.79)	(5.65)	(5.37)	(3.97)	(3.45)
$\alpha_t q_t$	-3.35***	-4.32***	-4.21***	-2.20**	-3.68***	-3.57***	-3.90***	-4.76***
	(-2.97)	(-4.93)	(-4.10)	(-2.40)	(-3.89)	(-3.70)	(-4.29)	(-4.37)
$sentiment_t$	-0.64							0.26
	(-0.94)							(0.39)
$disaster\ probabilities_t$		1.93						6.67***
		(3.30)						(0.97)
$ambiguity_t$			-1.06					-0.08
			(-1.61)					(-0.09)
$PLS\ disagreement_t$				-1.84***				-1.23*
				(-3.25)				(-1.75)
$analyst\ disagreement_t$					-0.60			-2.88***
					(-1.02)			(-4.20)
$average\ skewness_t$						-0.99***		-0.98**
						(-2.60)		(-2.39)
$risk\ aversion_t$							0.41	-4.82***
							(0.32)	(-2.87)
N	389	319	344	347	383	383	392	292
adj. R^2	0.110	0.133	0.124	0.136	0.106	0.117	0.107	0.263
Δ adj. R^2	0.054	0.120	0.113	0.022	0.090	0.083	0.098	0.065

Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium ($r^e_{[t+1,t+h]}$) in percent, against the conditional stock market volatility (q_t) and the product of market volatility and market ambiguity attitude ($\alpha_t q_t$) controlling for market sentiment (Baker and Wurgler, 2006), time-varying disaster probabilities (Barro and Liao, 2021), market ambiguity (Brenner and Izhakian, 2018), the partial least squares (PLS) disagreement index (Huang et al., 2021), analyst disagreement (Yu, 2011), value-weighted average skewness (Jondeau et al., 2019), and time-varying risk aversion (Bekaert et al., 2022). The top and bottom panels display results at the monthly ($h = 1$) horizon and the quarterly ($h = 3$) horizon. For ease of interpreting coefficients, each variable is divided by its unconditional standard deviation over the subset of our sample period (1990:01 - 2022:12) for which data is available. The number (N) of observations in each regression, the adjusted R^2 and the increase in adjusted R^2 (Δ adj. R^2) from including $\alpha_t q_t$ are also shown.

Figure 2. OOS Equity Premium Prediction with Volatility and Optimism (One-Month Forecast)



Notes: This figure displays the difference in cumulative sum of squared errors, $\Delta CSSE_{OOS}$, between the one-month-ahead forecast of the log equity premium based on the historical average and the one-month-ahead forecast based on (i) the conditional market volatility from a GARCH(1,1) model (from Section 2.6) in the left panel and (ii) the product of the conditional market volatility and the conditional market ambiguity attitude in the right panel. The out-of-sample period spans the second half of the sample, 2006:07 - 2022:12. Shaded periods are NBER recessions.

Table 5. R^2_{OS} for the Risk-Return Tradeoff

	Monthly Horizon		Quarterly Horizon		Six-Month Horizon		Annual Horizon	
Predictors	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW
q_t	-0.16	-0.44	-1.68	-1.07	-1.73	-1.78	-8.42	-3.83***
q_t, $\alpha_t q_t$	4.06	2.55**	12.73	4.03***	19.14	4.23***	11.07	4.53***

Notes: The table displays the Campbell and Thompson (2008) R^2_{OS} statistic (in percent) for predictor variables at the monthly, quarterly, six-month, and annual forecast horizons of the log equity premium. The dependent variable is, respectively, the one-month, cumulative three-month, cumulative six-month, and cumulative twelve-month log equity premium. The sets of predictors are market volatility (q_t), and volatility and the product of volatility and ambiguity attitude (q_t, $\alpha_t q_t$). CW is the Clark and West (2007) MSPE-adjusted statistic. ** and *** denotes significance at the 5%, and 1% levels. The out-of-sample period spans the second half of the sample, 2006:07 - 2022:12.

Table 5 displays the R^2_{OS} statistics for the out-of-sample predictive regressions based on q_t and $\{q_t, \alpha_t q_t\}$ as predictors. From Table 5, we see that market volatility alone does not have out-of-sample predictive power, with a negative R^2_{OS} at each horizon. In contrast, q_t and $\alpha_t q_t$ jointly produce a positive R^2_{OS} statistic above 4% at the one-month horizon, which increases to 12.73% for cumulative three-month returns and to 19.14% for cumulative six-month returns. That α_t, when combined with market volatility, produces such large increases in out-of-sample predictability reinforces our conclusion that α_t is a missing state variable that restores the risk-return tradeoff.

Figure 2 plots the evolution of the forecast performance of market volatility (left panel) and of market volatility and the product of ambiguity attitude and volatility (right panel) over the out-of-sample period (2006:07 - 2022:12). The difference between the two panels is striking. Market volatility has greater cumulative sum of squared forecast errors than the historical average equity premium forecast throughout the sample period. In contrast, the forecast in the right panel has a consistent upward trend, indicating our model forecasts are outperforming the benchmark.

4 Market Ambiguity Attitude, Market Crashes, and Recessions

We next examine the link between market ambiguity attitude, stock market fluctuations, and business cycles. Under the theory in Section 2, higher market ambiguity attitude reflects a more over-valued market relative to the expected utility benchmark. In the Markov-switching model, a high α_t also coincides with a regime in which $\frac{\mathrm{EP}_t}{q_t \gamma_t}$ is low, which corresponds on average to low expected market excess returns and high conditional market volatility. Large market declines are natural consequences of a regime with low expected returns and high volatility.

The predictive ability of α_t might also reflect time-varying macroeconomic risk linked to the real economy. If α_t is mean-reverting, and if the economy slows down due to a decline in the representative agent's α_t (i.e., if a decline in optimism reduces consumption expenditures by consumers and investment by firms), then high α_t could positively forecast recessions. We consider these two channels through which α_t might predict future returns. Our tests confirm that α_t predicts stock market fluctuations (returns and crashes) and business cycle fluctuations (recessions).

4.1 Market Ambiguity Attitude and Market Crashes

Panel A of Table 6 provides evidence that a high level of α_t systematically precedes large market declines. The table shows the frequency of large market declines (one-month market returns below -10% (top row) and below -5% (bottom row) in three sample cases. For the full sample, there were six market crashes of at least 10%, that occurred in roughly 1.5% of the periods, and 39 market crashes of at least 5% that occurred in approximately 10% of the periods. The second column of Table 6 is the frequency of crashes that occurred in periods in which α_t was in the top 33% of α_t values within the preceding three months (across the full sample of α_t values). The table shows that a high level of market ambiguity attitude in the three months prior to a given period t increases

Table 6. Frequency of Market Crashes and Recessions

	Frequency of Market Crashes			Crashes Predicted
Panel A	Unconditional	α (Top 33%)	α (Bottom 67%)	α (Top 33%)
10% Market Declines	1.53% (6)	4.14% (6)	0.00% (0)	100.00% (6)
5% Market Declines	9.95% (39)	19.31% (28)	4.45% (11)	71.79% (28)
	Frequency of NBER Recession Periods			**Recessions Predicted**
Panel B	Unconditional	α (Top 33%)	α (Bottom 67%)	α (Top 33%)
NBER Recessions	9.18% (36)	17.93% (26)	4.05% (10)	72.22% (26)

Notes: Panel A (Panel B) displays the frequency of large market declines (NBER recessions) in percent, with the total number in parentheses, across the (i) full sample period, denoted "Unconditional"; (ii) across periods in which α surpassed the top 33% of full-sample α values within the preceding three months; (iii) across periods in which α did not surpass the top 33% of α values within the preceding three months. The fourth column displays the proportion of realized crashes (recessions) that occurred in a period in which α surpassed the top 33% of α values within the preceding three months. The first and second rows of Panel A display the results for one-month declines in the market exceeding 10% and exceeding 5%, respectively. The data covers the period from 1990:01 - 2022:12.

the frequency of 10% crashes in period t to above 4%, more than double the unconditional average. The frequency of 5% crashes also roughly doubles to nearly 20%. In contrast, none of the 10% market declines and less than five percent of the 5% market declines occurred in periods in which α_t was not in the top 33% of α_t values in the preceding three months. The fourth column in Table 6 presents the frequency of a high level of α_t in the preceding three months, given a crash occurred in period t. All six crashes of at least 10% occurred in periods in which α_t was in the top 33% of α_t values in the previous three months. The bottom row shows that roughly 72% of all 5% crashes occurred in periods in which α_t was in the top 33% of all α_t values in the previous three months.

Table 7 uses logistic regressions to test whether α_t predicts 5% or 10% market crashes. The left-hand-side variable is an indicator of either a 10% crash or 5% crash. Our baseline specification summarized in column (1) (for a 10% crash) and column (7) (for a 5% crash) includes only α_t on the right-hand-side (lagged three months). The remaining columns include controls (q_t, VIX, pd), each lagged three months, which are used in the construction of α_t and the measure of time-varying risk aversion, ra_t, from Bekaert et al. (2022). These variables are plausible predictors of a market crash. Table 7 shows α_t is a significant predictor of both 10% and 5% crashes at the quarterly horizon and that its predictive power is not subsumed by q, VIX, pd, or ra_t.

Table 7. Predicting Market Crashes with Market Ambiguity Attitude

	Logistic Regressions for Predicting Market Crashes											
	(1) -10%	(2) -10%	(3) -10%	(4) -10%	(5) -10%	(6) -10%	(7) -5%	(8) -5%	(9) -5%	(10) -5%	(11) -5%	(12) -5%
α_{t-3}	1.65***	1.58***	1.58***	2.38***	1.60***	3.24***	0.77***	0.81***	0.81***	0.83***	0.71***	1.17***
	(4.06)	(3.47)	(3.15)	(3.60)	(3.39)	(5.43)	(4.19)	(3.89)	(3.60)	(3.14)	(3.71)	(3.45)
q_{t-3}		0.31				0.01		-0.08				-0.25
		(0.46)				(0.02)		(-0.38)				(-1.11)
VIX_{t-3}			0.17			-2.85**			-0.07			-1.69***
			(0.27)			(-2.44)			(-0.26)			(-3.31)
pd_{t-3}				-0.69		-0.41				-0.08		0.12
				(-1.15)		(-0.64)				(-0.35)		(0.49)
ra_{t-3}					0.32	2.18**					0.16	1.55***
					(1.18)	(2.13)					(1.14)	(3.58)
Pseudo R^2	0.164	0.171	0.167	0.199	0.189	0.285	0.079	0.080	0.080	0.080	0.084	0.134

Robust Z statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays the slope coefficients from logistic regressions. In columns (1) - (6), the left-hand-side variable equals one in period t if a market return less than -10% occurred in period t, and zero otherwise. In columns (7) - (12), the left-hand-side variable equals one in period t if a market return less than -5% occurred in period t, and zero otherwise. The right-hand-side variables (each lagged three months) are the market ambiguity attitude. α, the conditional market volatility, q, the VIX index of the Chicago Board of Options Exchange, the price-dividend ratio, pd, of the S&P 500 index, and the measure of time-varying risk aversion, ra, from Bekaert et al. (2022). The results are shown for the full sample period (1990:01 - 2022:12). For convenience in interpreting the coefficients, each right-hand-side variables is divided by its full-sample standard deviation.

4.2 Market Ambiguity Attitude and NBER Recessions

Table 6, Panel B, shows that 72% of all recession periods across our sample period occur when α is within the top 33% of α values in the preceding three months. At such times, the unconditional frequency of NBER recessions for our sample period (9.18%) nearly doubles to 17.93%.

Table 8 summarizes logistic regressions with α as a predictor variable for recessions at the three-month horizon with various sets of control variables.[15][16] Table 8 shows α significantly predicts recessions across each set of control variables. Adding α to regression specification (7) with all eight control variables increases the Pseudo R^2 by 11 percentage points.

[15] Similar results are obtained using probit regressions.

[16] Liu and Moench (2016) identify the term spread and the aggregate stock market return as the two strongest recession predictors at short horizons including the three-month horizon. Guha and Hiris (2002) find that credit spreads also predict recessions. We thus include as controls the term spread, TMS (the difference between the long-term yield on U.S. government bonds and the U.S. treasury bill), the aggregate stock market return, R_m, and the default yield spread, DFY (the difference between BAA and AAA-rated corporate bond yields). These variable each significantly predict recessions over our sample period. We also include the variables used in the construction of α (q, VIX, and pd), along with the Baker and Wurgler (2006) sentiment index, the Bekaert et al. (2022) risk aversion index, and the lagged NBER recession indicator. In regression specification (6), both α and risk aversion positively and significantly predict recessions, although risk aversion in not significant in the kitchen sink specification in (7).

Table 8. Predicting Recessions with Market Ambiguity Attitude

Logistic Regressions for Predicting Recessions							
	(1) REC	(2) REC	(3) REC	(4) REC	(5) REC	(6) REC	(7) REC
α_{t-3}	0.85***	0.79***	0.77***	1.95***	0.79***	0.65***	3.29***
	(4.68)	(3.18)	(3.00)	(6.06)	(4.46)	(3.16)	(4.04)
Lagged Recession Predictors	NO	YES	NO	NO	NO	NO	YES
Lagged Recession Indicator	NO	NO	YES	NO	NO	NO	YES
Lagged α Ingredients	NO	NO	NO	YES	NO	NO	YES
Lagged Sentiment Index	NO	NO	NO	NO	YES	NO	YES
Lagged Risk Aversion Index	NO	NO	NO	NO	NO	YES	YES
Pseudo R^2	0.093	0.277	0.448	0.288	0.099	0.171	0.682
Δ(Pseudo R^2)	0.093	0.051	0.040	0.134	0.066	0.043	0.110

Robust z statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays the coefficients from logistic regressions. The left-hand-side variable is the NBER recession indicator (REC), which equals one in period t if there was a recession in period t and equals zero otherwise. The right-hand-side variables (lagged three months) include α and eight control variables: three "recession predictors" (the term spread, TMS, the default yield spread, DFY, and the aggregate stock market return, R_m); the three "α ingredients" (q, VIX, and pd); Baker and Wurgler (2006) sentiment index; Bekaert et al. (2022) risk aversion index; and the lagged NBER recession indicator. The table covers the period from 1990:01 - 2022:12, except in regression specification (2) which ends in 2021:12, specification (5) which ends in 2022:06, and specification (6) which ends in 2021:12 due to data availability. Δ(Pseudo R^2) denotes the change in Pseudo R^2 from including α in the regression relative to an otherwise identical regression that excludes α. For convenience in interpreting the coefficients, α is divided by its full-sample standard deviation.

5 The Value of Information in Market Ambiguity Attitude

Suppose the equity premium is increasing in q_t and decreasing in $\alpha_t q_t$ as our results suggest. Consider a setting with a textbook mean-variance investor who cannot affect prices and wants to construct a dynamic portfolio that exploits the predictive power in q_t and α_t. How does such an investment strategy perform relative to that of an agent who trades assuming the equity premium is increasing in q_t (neglecting a role for α_t) or who adopts a buy-and-hold strategy, passively holding the market portfolio? To investigate this, we construct an out-of-sample trading strategy using the classical Merton (1969) investment and asset allocation model. Doing so provides a strong out-of-sample test of the quality and value of forecasts generated by q_t and $\alpha_t q_t$.

We next consider the investment performance of a portfolio that uses the equity premium forecasts generated by q_t and $\alpha_t q_t$. As discussed by Ferreira and Santa-Clara (2011), Jondeau et al.

(2019), and Giglio et al. (2021), we use the formula for Markowitz optimal weight on the market portfolio, $w_t = [E_t[R_{t+1}] - R_{f,t}]/[\lambda q_t^2]$. Following Jondeau et al. (2019) we set the risk aversion parameter to $\lambda = 2$ and add the realistic portfolio constraint that $w_t \in [0, 2]$ which excludes short-selling and permits at most 100% leverage. The ex post portfolio excess return, $R^e_{p,t+1}$, at the end of month $t+1$ is then $R^e_{p,t+1} = w_t \, R^e_{m,t+1}$, where $R^e_{m,t+1}$ denotes the market excess return in period $t+1$. As noted by Jondeau et al. (2019), repeating this process for each period from the first out-of-sample period through the end of the sample period, yields a time series of ex post excess returns for each optimal portfolio. We will evaluate the performance of each portfolio according to the portfolio's realized Sharpe ratio during the out-of-sample period and the certainty equivalent return on portfolio p for a mean-variance investor, defined as $CER = \overline{R}_p - (\lambda/2)\sigma^2_p$, where σ^2_p is the variance of the portfolio return. This quantity is the risk-free return that would make a mean-variance investor with risk aversion λ indifferent between that return and investing in portfolio p. We also test if the investment strategies earn significant risk-adjusted returns relative to the Fama and French (2018) six factor model and the Hou et al. (2021) five-factor q-factor model.

We consider three equity premium forecasts at the one month horizon. The sets of predictors used to generate the forecasts are: (i) q_t and $\alpha_t q_t$; (ii) q_t; and (iii) the historical average forecast. As a benchmark, we also consider a fourth investment strategy, the buy-and-hold strategy of passively holding the market portfolio. To compute the conditional variance in the optimal portfolio weight, we use q_t^2, the conditional market variance from the GARCH(1,1) model in Section 2.6. Table 9 summarizes the investment performance with strategies ranked by their realized Sharpe ratio.

As shown in the Table 9, the investment strategy based on q_t and $\alpha_t q_t$ is the only one to earn significant abnormal returns relative to the Fama-French six factor and the q five factor models. Neither the investment strategy based on market volatility, q_t, alone, or the strategy based on the historical average outperforms the passive buy-and-hold strategy in terms of the portfolio Sharpe ratio or certainty equivalent return. The strategy that combines α_t with q_t generates a Sharpe ratio that is 39% higher than that of the passive buy-and-hold strategy (0.78 versus 0.56) and a CER that is roughly double that of the buy-and-hold strategy. One might view the ratio of the strategy's Sharpe ratio to the market Sharpe ratio or the difference between the strategy's CER and the market CER as a measure of the value of the information contained in α_t.

Table 9. Out-of-Sample Investment Performance

Predictors	$\bar{w}$	Ret	Vol	SR	CER	α^{FF6}	α^{q5}
q, αq	1.31	1.46	6.51	0.78	13.17	7.06**	7.96**
Buy-and-hold	1.00	0.76	4.69	0.56	6.68	0.00	0.00
q	1.51	0.87	6.71	0.45	5.16	-2.14	-0.12
Historical avg.	1.42	0.71	6.14	0.40	4.07	-2.26	-1.31

Notes: The table displays the out-of-sample performance of investment strategies that update the weights on the market portfolio based on forecasts of the equity premium at the one month forecast horizon. The weight on the market portfolio in each period is the one-month-ahead equity premium forecast divided by the product of the coefficient of relative risk aversion (λ) and the conditional market variance (q_t^2). We set $\lambda = 2$ as suggested by Jondeau et al. (2019). The investment strategies correspond to forecasts based on (i) q_t, and the product $\alpha_t q_t$; (ii) q_t; (iii) the historical average forecast; and (iv) the passive strategy that buys and holds the market portfolio. The table displays the average weight on the market portfolio ($\bar{w}$), the average monthly return (Ret), the average monthly volatility (Vol) of the portfolio return, the annualized monthly Sharpe ratio (SR), the annualized certainty equivalent return for a mean-variance investor with $\lambda = 2$ (CER), and the annualized risk-adjusted returns relative to the Fama and French (2018) six factor model (α^{FF6}), and the Hou et al. (2021) five-factor q-factor model (α^{q5}). Returns are in percent. The data spans the out-of-sample period from 2006:07, through 2022:12. ** denotes the 5% level of statistical significance.

6 Robustness Checks and Extensions

We perform various analyses to evaluate the robustness of our results: (i) We test if the regression coefficients are stable across the two halves of the sample period. (ii) We test whether α_t restores the risk-return tradeoff in both halves of the sample period. (iii) We test if the results hold using alternative GARCH volatility models. (iv) We test if α_t has predictive power in the absence of market volatility and assess its predictive power over time. (v) We conduct additional tests to evaluate if the predictive power of q_t and $\alpha_t q_t$ holds at the longer six-month and twelve-month horizons. (vi) We test the performance of the log-linearized version of Equation (11) in which the log equity premium is approximately a linear function of q_t, α_t, and γ_t. (vii) We test if the results for market crashes and NBER recessions hold for the out-of-sample period. (viii) We construct α using the equity approximation with risk aversion, using CRRA parameters $\lambda = 1$ (log utility) and $\lambda = 2$, and test whether the resulting α series restores the risk-return tradeoff in-sample and out-of-sample. Our results are robust in each case. This section presents the parameter stability tests and the predictive regressions for each sub-sample. The remaining robustness checks are presented in Internet Appendix C.

6.1 Stability of Regression Coefficients

Motivated by Welch and Goyal (2008), we investigate if the regression coefficients are stable across the two halves of our sample. Table 10 displays the in-sample regression coefficients across the two halves of the sample period. Table 10 reveals that the regression with only q_t is unstable, as it changes sign from negative to positive. In contrast, the estimated coefficients for q_t are noticeably more stable when the interaction term $\alpha_t q_t$ is included in the regression. Further, the coefficients for $\alpha_t q_t$ are similar and not significantly different across the two halves of the sample period. For example, at the quarterly horizon, the coefficient estimates on market volatility are -0.58 and 1.51 in the two halves of the sample when only q_t is included. Including the interaction term $\alpha_t q_t$ in the regression yields estimated coefficients for q_t of 4.29 and 4.15 in the two halves of the sample and they are not significantly different. The coefficients for $\alpha_t q_t$ are -4.01 and -4.43 across the two halves of the sample and are also not significantly different. These observations further suggest α_t is a missing state variable that helps produce more stable forecasts of the risk-return tradeoff.

Table 10. Stability of Coefficients for the Risk-Return Tradeoff

		Monthly		Quarterly	
x_t	z_t	β_1	β_2	β_1	β_2
q_t		-0.03	0.43	-0.58	1.51
q_t	$\alpha_t q_t$	1.78	1.43	4.29	4.15
$\alpha_t q_t$	q_t	-1.35	-1.69	-4.01	-4.43

Notes: The table displays coefficients from full-sample predictability regressions for the first and second half of the sample period for the monthly and quarterly forecast horizons. The first sub-sample spans monthly data from 1990:01 - 2006:06. The second sub-sample spans from 2006:07 - 2022:12. β_1 and β_2 denote the estimated coefficients for the first and second halves of the sample. They are estimated from the regression $r^e_{[t+1,t+h]} = a + \beta\, x_t + \beta_D\, D + \beta_{Dx}\, D\, x_t + \beta_z z_t + \epsilon_{[t+1,t+h]}$ where D is a dummy variable that equals 0 in the first half of the sample period and 1 in the second half of the sample, $h \in \{1, 3\}$, $\beta_1 \coloneqq \beta$ and $\beta_2 \coloneqq \beta + \beta_{Dx}$. The predictor variables include market ambiguity attitude, α_t, conditional market volatility, q_t, measured from a GARCH(1,1) model, and the product $\alpha_t q_t$. For ease of interpreting the coefficients, q_t and $\alpha_t q_t$ are divided by their (full sample) standard deviation.

6.2 Performance Across Subsamples

Table 11 (columns (6) and (9) in both Panels A and B) shows that α_t restores the risk-return tradeoff for both halves of the sample period at both the monthly and quarterly horizons. In each case, the significantly positive risk-return relation is recovered when $\alpha_t q_t$ is included in the regression with q_t. Further, including both q_t and $\alpha_t q_t$ more than triples the R^2, relative to just including q_t for each of regressions (3), (6), and (9) at both the monthly and quarterly horizons.

Table 11. Market Ambiguity Attitude and the Risk-Return Tradeoff Across Subsamples

Monthly	Full Sample (1990 - 2022)			Out-of-Sample Period			Training Sample Period		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}
q_t	0.30		1.35***	0.43		1.51***	-0.02		1.56**
	(1.18)		(5.44)	(1.38)		(5.45)	(-0.04)		(2.29)
$\alpha_t q_t$		-0.32	-1.36***		-0.28	-1.69**		-0.35	-1.37**
		(-1.07)	(-3.87)		(-0.55)	(-2.59)		(-1.02)	(-2.35)
R^2	0.005	0.005	0.042	0.011	0.003	0.061	0.000	0.007	0.027
Quarterly	**Full Sample (1990 - 2022)**			**Out-of-Sample Period**			**Training Sample Period**		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel B	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$
q_t	0.92		3.89***	1.50**		4.40***	-0.51		3.65**
	(1.38)		(5.63)	(2.02)		(5.78)	(-0.40)		(2.28)
$\alpha_t q_t$		-0.88	-3.86***		-0.49	-4.58***		-1.22	-3.60**
		(-1.13)	(-4.07)		(-0.39)	(-2.90)		(-1.25)	(-2.42)
R^2	0.014	0.012	0.112	0.046	0.003	0.165	0.003	0.028	0.062

Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium, $r^e_{[t+1,t+h]}$, in percent, against the conditional stock market volatility (q_t), estimated from a GARCH(1,1) model (from Section 2.6), in regression specifications (1), (4), and (7); regressions of the log equity premium against the product of market ambiguity attitude and the conditional market volatility ($\alpha_t q_t$) in regression specifications (2), (5), and (8); and regressions of the log equity premium against both variables in regression specifications (3), (6), and (9). Formally, we run versions of the following regression that include one or both of the right-hand-side variables:

$$r^e_{[t+1,t+h]} = \beta_0 + \beta_1 q_t + \beta_2 \alpha_t q_t + \epsilon_{[t+1,t+h]}. \tag{22}$$

Regressions (1), (2), and (3) span monthly data from the full sample period (1990:01 - 2022:12). Regressions (4), (5), and (6) use data from the second half of this sample (2006:07 - 2022:12) which is the period for which q_t and α_t are recursively estimated using only information available to investors in real time. Regressions (7), (8), and (9) use data from the first half of the sample period (1990:01 - 2006:06) which served as the training period in which q_t and α_t were estimated using all data in the first half of the sample. Regressions are over a forecast horizon of $h = 1$ month (monthly horizon) in Panel A and over a forecast horizon of $h = 3$ months (quarterly horizon) in Panel B.

7 Conclusion

This paper studies the effect of market ambiguity attitude on the risk-return tradeoff. We consider a representative agent asset pricing model in which equilibrium prices depend on an information component (reflecting the asset's discounted expected value) and an ambiguity attitude component. The equilibrium equity premium depends on market optimism, Knightian uncertainty, positive skewness, and disaster risk, linking these strands of the asset pricing literature. Our model yields the theoretical implication that the equity premium is increasing in market volatility and the slope of this relationship flattens as market ambiguity attitude increases. We develop a theory-based measure of the market's ambiguity attitude and test the theoretical implication that it predicts time variation in the market risk-return tradeoff. We also test if market ambiguity attitude predicts market crashes and recessions.

Our paper adds to the literature on applications of ambiguity models to finance and to the literature on time-variation in the equity premium (Campbell and Cochrane, 1999; Cohn et al., 2015) by identifying a new source of time-varying expected returns which we have shown has distinct predictive power from market sentiment, ambiguity, skewness, disaster risk, disagreement, time-varying risk aversion, and 25 standard variables used to predict the equity premium.

We find that the predicted positive relationship between the equity premium and the conditional market volatility is observed only after accounting for the market ambiguity attitude. This finding holds both in-sample and out-of-sample, at both the monthly and quarterly forecast horizons, and it is not subsumed by market sentiment or established equity premium predictors. Our paper documents that market ambiguity attitude predicts market crashes, consistent with high levels of optimism reflecting an over-valued market relative to an expected utility representative agent. Further, market ambiguity attitude predicts NBER recessions, indicating that market ambiguity attitude provides a link between stock market and business cycle fluctuations. The information in market ambiguity attitude substantially increases the Sharpe ratio and certainty equivalent return of a mean-variance investor relative to using only market volatility as a predictive signal, or to adopting a buy-and-hold strategy. Our results indicate that market ambiguity attitude is an important state variable in driving time-varying expected returns, and might help to bridge the gap between irrational exuberance in the stock market and equilibrium asset pricing theory.

Appendix

Proof of Proposition 1: The equity premium from Equation (8) is the following

$$EP_t = -\frac{\mathrm{Cov}_t(M_{t+1}, R_{t+1})}{E_t M_{t+1}} + \frac{\gamma_t}{1-\gamma_t}\Big(\alpha_t \frac{\overline{M}_{t+1}}{E_t M_{t+1}}(R_{f,t} - \overline{R}_{t+1}) + (1-\alpha_t)\frac{\underline{M}_{t+1}}{E_t M_{t+1}}(R_{f,t} - \underline{R}_{t+1})\Big),$$

where $EP_t := R_{t+1} - R_{f,t}$, and $M_{t+1} := \delta\frac{u'(C_{t+1})}{u'(C_t)}$. Recall that $\overline{M}_{t+1}$ and $\underline{M}_{t+1}$ are associated with the next period's optimistic and pessimistic consumption growth rates. Given the CRRA utility function, $M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\lambda} = e^{-\rho-\lambda\Delta c_{t+1}}$, where $\rho := -\ln\delta$. The normality of log consumption growth Δc_{t+1} in (9) implies $E_t M_{t+1} = e^{-\rho-\lambda g+\frac{1}{2}\lambda^2\sigma^2}$.[17] From Assumption 1, we have $\overline{M}_{t+1} = e^{-\rho-\lambda g-\lambda\overline{\xi}\sigma}$, and $\underline{M}_{t+1} = e^{-\rho-\lambda g+\lambda\underline{\xi}\sigma}$. Similarly, from the normality of log returns and Assumption 1, we have $\overline{R}_{t+1} = e^{\mu_t+\overline{\xi}q_t}$, and $\underline{R}_{t+1} = e^{\mu_t-\underline{\xi}q_t}$. Finally, the covariance term in the EP_t equation is the following

$$\mathrm{Cov}_t(M_{t+1}, R_{t+1}) = E_t M_{t+1}R_{t+1} - E_t M_{t+1}E_t R_{t+1} = e^{-\rho-\lambda g+\frac{1}{2}\lambda^2\sigma^2+\mu_t+\frac{1}{2}q_t^2}\big(e^{-\lambda\eta\sigma q_t} - 1\big).$$

Substituting $E_t M_{t+1}$, $\overline{M}_{t+1}$, $\underline{M}_{t+1}$, $\overline{R_{t+1}}$, $\underline{R_{t+1}}$, and the covariance term, we have

$$\begin{aligned} EP_t &= e^{\mu_t+\frac{1}{2}q_t^2}\left(1 - e^{-\eta\lambda\sigma q_t}\right) + \\ &\frac{\gamma_t\alpha_t}{1-\gamma_t}\left(e^{-\lambda\overline{\xi}\sigma-\frac{1}{2}\lambda^2\sigma^2}\left(R_{f,t} - e^{\mu_t+\overline{\xi}q_t}\right) + (1-\alpha_t)e^{\lambda\underline{\xi}\sigma-\frac{1}{2}\lambda^2\sigma^2}\left(R_{f,t} - e^{\mu_t-\underline{\xi}q_t}\right)\right). \end{aligned} \tag{23}$$

We use the approximation $e^x \approx 1 + x$, which is accurate for small values of x. Note that since $E_t R_{t+1} \approx 1 + \mu_t + \frac{1}{2}q_t^2$, we have $1 + \mu_t - R_{f,t} \approx EP_t - \frac{1}{2}q_t^2$. Thus, we can rewrite the above as

$$\begin{aligned} &EP_t \approx \eta\lambda\sigma q_t\left(1 + \mu_t + \frac{1}{2}q_t^2\right) - \\ &\frac{\gamma_t}{1-\gamma_t}\left(\alpha_t\Big(1 - \lambda\overline{\xi}\sigma - \frac{1}{2}\lambda^2\sigma^2\Big)\left(EP_t + \overline{\xi}q_t - \frac{1}{2}q_t^2\right) + (1-\alpha_t)\Big(1 + \lambda\underline{\xi}\sigma - \frac{1}{2}\lambda^2\sigma^2\Big)\left(EP_t - \underline{\xi}q_t - \frac{1}{2}q_t^2\right)\right). \end{aligned} \tag{24}$$

The covariance term is the expected market return, multiplied by $\lambda\eta\sigma q_t$; however, using the standard monthly calibration $\sigma = 0.016/\sqrt{12}$, $q = 0.16/\sqrt{12}$, $\eta = 0.2$, and with log utility $\lambda = 1$ as the baseline, this term contributes about five basis points annually to the equity premium, which is negligible. Moreover, in comparison to the first-order terms, the second-order terms $\frac{1}{2}q_t^2$ and $\frac{1}{2}\lambda^2\sigma^2$ are negligible, which we drop. Next, we use $\frac{1}{1+x} \approx 1 - x$, which is accurate for small values of x,

[17] Recall that if $x \sim \mathcal{N}(\mu, \sigma^2)$, then $E\,e^x = e^{\mu+\frac{1}{2}\sigma^2}$.

and replace $\overline{\xi}$ and $\underline{\xi}$ with ξ, to find

$$EP_t \approx \Big(1 - 2\alpha_t + \lambda\sigma\xi\big(1 - (1-2\alpha_t)^2\gamma_t\big)\Big)\xi\gamma_t q_t.$$

Finally, note that the term $(1-2\alpha_t)^2\gamma_t$ is, on average, about 0.01 (since the mean values of α_t and γ_t are about 27% and 5.0%, respectively), and hence, the term $-\lambda\sigma\xi(1-2\alpha_t)^2\gamma_t$ in the parenthesis is negligible. Thus, we find the approximation

$$EP_t \approx \big(1 - 2\alpha_t + \lambda\sigma\xi\big)\xi\gamma_t q_t. \tag{25}$$

A brief discussion about the accuracy of the EP_t approximation formula (25) is in order. We show that solving EP_t directly from (24) gives numerically an almost identical value to the approximation. We use the log utility parameter $\lambda = 1$ as our baseline. First, using the (monthly) average values, i.e., $\sigma = 0.016/\sqrt{12}$, $q = 0.16/\sqrt{12}$, $\eta = 0.2$, $r_f = 0.01/12$, $\alpha = 0.27$, and $\gamma = 0.050$, and the parameter $\xi = 4.77$, the EP values from (24) and (25) are 0.00535 and 0.00531, respectively. Second, using our estimated series of α_t and γ_t, and the monthly data series for q_t, and $r_{f,t}$, and the parameter values $\sigma = 0.016/\sqrt{12}$, $\eta = 0.2$, and $\xi = 4.77$, the correlation between the time variable EP_t from (24) and (25) is 0.999. They are graphed in Figure 3 in Internet Appendix B. When plotting the two series together, the difference is hardly visible.

Proof of Proposition 2: We start with the definition $VRP_t \coloneqq \mathrm{Var}_t^Q R_{t+1} - \mathrm{Var}_t R_{t+1}$. Note that the conditional log normality of returns implies that $\sigma_t(R_{t+1}) = E_t R_{t+1}\sqrt{e^{q_t^2} - 1} \approx (1 + \mu_t + \frac{1}{2}q_t^2)q_t \approx q_t$. That is, the log returns' and returns' conditional variance are approximately the same. Next, using the fact that under the risk-neutral measure, the expected market return equals the risk-free rate, we have

$$\begin{aligned} VRP_t = {} & (1-\gamma_t)E_t R_{f,t} M_{t+1}(R_{t+1} - R_{f,t})^2 + \\ & \gamma_t\Big(\alpha_t R_{f,t}\overline{M}_{t+1}(\overline{R}_{t+1} - R_{f,t})^2 + (1-\alpha_t)R_{f,t}\underline{M}_{t+1}(\underline{R}_{t+1} - R_{f,t})^2\Big) - q_t^2. \end{aligned} \tag{26}$$

Similar to the EP_t approximation, the contribution of the covariance term (between M_{t+1} and R_{t+1}^2) is negligible. This implies that $E_t R_{f,t} M_{t+1}(R_{t+1} - R_{f,t})^2 \approx q_t^2$, which is not surprising, as it is well-known that the CRRA utility creates no variance risk premium. Further, $R_{f,t}\overline{M}_{t+1} \approx 1 - \lambda\overline{\xi}\sigma$, and

$R_{f,t}\underline{M}_{t+1} \approx 1 + \lambda\underline{\xi}\sigma$. Thus, we have

$$VRP_t \approx \gamma_t\alpha_t\Big(1 - \lambda\bar{\xi}\sigma\Big)\left(EP_t + \bar{\xi}q_t\right)^2 + \gamma_t(1-\alpha_t)\Big(1 + \lambda\underline{\xi}\sigma\Big)\left(EP_t - \underline{\xi}q_t\right)^2 - \gamma_t q_{t+1}^2.$$

Replacing $EP_t \approx \big(1 - 2\alpha_t + \lambda\sigma\xi\big)\xi\gamma_t q_t$ from the EP_t approximation and using $\bar{\xi} = \underline{\xi}$, after collecting terms, the previous expression becomes

$$VRP_t \approx \Big[1 - (2\gamma_t - \gamma_t^2)(1-2\alpha_t)^2 + (1-2\alpha_t)\big(1 - 2\gamma_t + (1-2\alpha_t)^2\gamma_t^2\big)\xi\lambda\sigma\Big]\xi^2 q_t^2\gamma_t - \gamma_t q_t^2. \tag{27}$$

The term inside the bracket is approximately one. For instance, with the log utility ($\lambda = 1$) as the baseline, and using the average (monthly) values $\sigma = 0.016/\sqrt{12}$, $\alpha = 0.27$, $\gamma = 0.050$, and the parameter $\xi = 4.77$, the two terms after one in the bracket are about -0.02 and 0.01 respectively, which makes their sum negligible compared to one. Thus, we find that

$$VRP_t \approx (\xi^2 - 1)q_t^2\gamma_t. \tag{28}$$

We present a brief demonstration of how accurately the VRP_t in (28) approximates (27). We use log utility ($\lambda = 1$) as the baseline. First, we compare the righ-hand sides of (27) and (28) at the average (monthly) values $\sigma = 0.016/\sqrt{12}$, $q = 0.16/\sqrt{12}$, $\alpha = 0.27$, $\gamma = 0.050$, and the parameter $\xi = 4.77$, to find the (monthly) VRP values of 0.00229 and 0.00232, respectively. Second, we look at the correlation between the right-hand sides of (27) and (28) using the (monthly) value of $\sigma = 0.016/\sqrt{12}$ and data series q_t, together with our estimated time series of α_t and γ_t with parameter $\xi = 4.77$. The correlation between the two VRP_t series is 0.999. They are graphed in Figure 4 in Internet Appendix B. We conclude that the approximation is quite accurate.

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Internet Appendix

Appendix A Data Appendix

This appendix contains the sources of data used in the paper.

1. **Market Excess Return:** The market excess return (Rm-Rf), market return (Rm), and risk-free rate (Rf) are from Kenneth French's data library: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

2. **Baker-Wurgler Sentiment Index (BW):** The Baker and Wurgler (2006) market sentiment index (*bw*) is from Jeffrey Wurgler's website: `https://pages.stern.nyu.edu/~jwurgler/`.

3. **Barro-Liao U.S. Disaster Probabilities:** The Barro and Liao (2021) U.S. disaster probability data series is available from Gordon Liao's website at: `https://gliao.xyz/research/`.

4. **Cederburg, Johnson, & O'Doherty Equity Premium Predictors:** The eleven predictors used from Cederburg et al. (2023) were shared with us by the authors of that paper. Their data extends through December, 2017. We were able to have data updated through 2021 for all eleven of the predictors in their paper that have data available at the start of our sample period (January, 1990). The eleven predictors are: West Texas Intermediate oil price changes (Driesprong et al., 2008), the variance risk premium (Bollerslev, 1986), the output gap (Cooper and Priestley, 2009), average correlation (Pollet and Wilson, 2010), nearness to the DOW all-time high (Li and Yu, 2012), new orders-to-shipments of durable goods (Jones and Tuzel, 2013), the tail-risk measure of Kelly and Jiang (2014), the PLS book-to-market factor (Kelly and Pruitt, 2013), short interest (Rapach et al., 2016), employment growth (Chen and Zhang, 2011), and the gold-to-platinum ratio (Huang and Kilic, 2019). Data extended through 2021 for the out-of-sample short interest index is available from Guofu Zhou's website at `http://apps.olin.wustl.edu/faculty/zhou/zpublications.html`. Data extended through 2021 for eight other predictors (West Texas Intermediate oil price changes, the variance risk premium, the output gap, average correlation, nearness to the DOW-all time high, new orders-to-shipments of durable goods, the tail-risk measure of Kelly and Jiang (2014), and the PLS book-to-market factor) were provided to us by Amit Goyal. The remaining two

series (employment growth and the gold-to-platinum ratio) were extended through 2021 in Azimi et al. (2023) using publicly available data according to the procedures described in the original papers (Chen and Zhang, 2011; Huang and Kilic, 2019).

5. **Goyal-Welch Equity Premium Predictors:** The 14 Goyal-Welch equity premium predictors at the monthly frequency are available from Amit Goyal's website: `https://sites.google.com/view/agoyal145`. The 14 equity premium predictors from Welch and Goyal (2008) that are available at the monthly frequency are the dividend price ratio (*dp*), the dividend yield (*dy*), the earnings price ratio (*ep*), the dividend payout ratio (*de*), realized stock market variance (*svar*), book-to-market ratio (*bm*), net equity expansion (*ntis*), treasury bill yield (*tbl*), long-term yield (*lty*), long-term treasury bond return (*ltr*), the term spread (*tms*), the corporate bond default yield spread (*dfy*), default return spread (*dfr*), and the consumer price index (*infl*).

6. **Ambiguity Index:** The ambiguity index from Brenner and Izhakian (2018) was provided to us directly by Yehuda Izhakian.

7. **PLS Disagreement Index:** The PLS disagreement index from Huang et al. (2021) is available on Dashan Huang's website at: `https://dashanhuang.weebly.com/`.

8. **Analyst Disagreement Index:** The analyst disagreement index from Yu (2011) was provided to us by Amit Goyal.

9. **Risk Aversion Index:** The time-varying risk aversion from Bekaert et al. (2022) is available from Nancy Xu's website at: `https://www.nancyxu.net/risk-aversion-index`.

10. **Short Interest Index:** The short interest index from Rapach et al. (2016) was provided to us directly by Guofu Zhou.

11. **NBER Recession Indicator:** The NBER recession indicator is from the St. Louis Federal Reserve Website (FRED), series USREC and is available at: `https://fred.stlouisfed.org/series/USREC`.

12. **Price Dividend Ratio (*pd*):** The price-dividend ratio (*pd*) of the S&P 500 index is computed as S&P composite price, P, divided by dividend D from Robert Shiller's website: `http:`

//www.econ.yale.edu/~shiller/data.htm.

13. **VIX and RNS:** The monthly VIX index and the marker Risk Neutral Skewness (RNS) are from the Chicago Board of Options Exchange (CBOE). Both are converted from daily to monthly series using the last index value for each month as the monthly value for that month. The daily VIX data is available at https://www.cboe.com/tradable_products/vix/vix_historical_data/. The daily SKEW index is available at https://www.cboe.com/us/indices/dashboard/skew/. RNS $= E[(\frac{R-\mu}{\sigma})^3]$, where R is the 30-day log-return on the S&P 500, μ and σ are respectively the mean and standard deviation of R, $x := (\frac{R-\mu}{\sigma})^3$ and RNS = E[x]. RNS is constructed from the SKEW index of the CBOE according to the relation: RNS = (100 - SKEW)/10. See the CBOE white paper on the SKEW index, page 5, at: https://cdn.cboe.com/resources/indices/documents/SKEWwhitepaperjan2011.pdf).

Appendix B Supplementary Tables and Figures

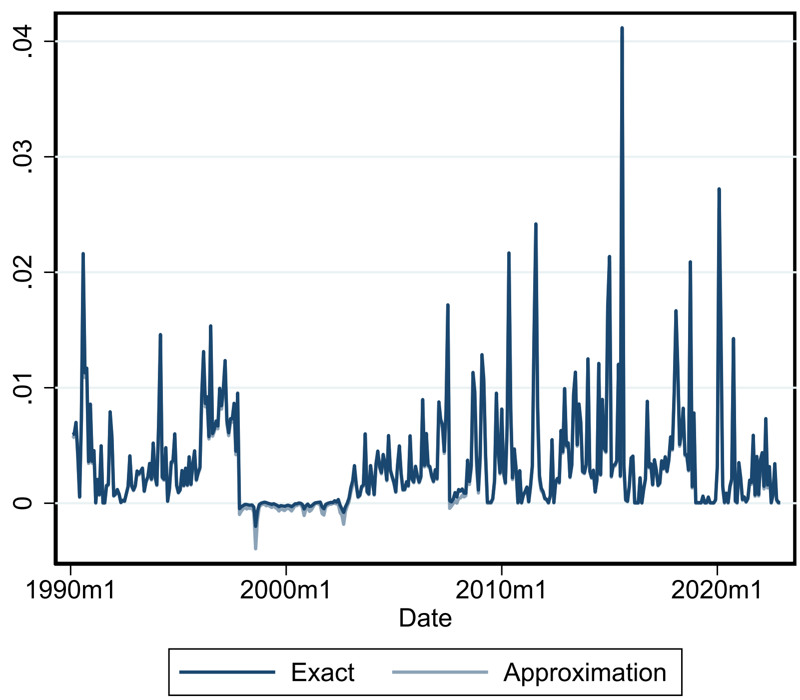
Table 12. GARCH(1,1) specifications of market return for up to 3 lags of the *pd* ratio.

	Full Sample (1990 - 2022)			Training Sample Period		
	(1)	(2)	(3)	(4)	(5)	(6)
pd_{t-1}	-0.037**			-0.033**		
	(0.015)			(0.017)		
pd_{t-2}		-0.037**			-0.032*	
		(0.015)			(0.017)	
pd_{t-3}			-0.036**			-0.033*
			(0.015)			(0.017)
Constant	2.838***	2.812***	2.744***	2.504***	2.45***	2.497***
	(0.827)	(0.834)	(0.831)	(0.916)	(0.930)	(0.935)
ARCH						
$ARCH_{t-1}$	0.192***	0.192***	0.195***	0.110	0.112	0.113
	(0.044)	(0.044)	(0.045)	(0.081)	(0.082)	(0.083)
$GARCH_{t-1}$	0.774***	0.774***	0.771***	0.864***	0.862***	0.860***
	(0.052)	(0.052)	(0.050)	(0.092)	(0.092)	(0.093)
Constant	0.985**	0.979**	0.998**	0.442	0.442	0.448
	(0.476)	(0.471)	(0.477)	(0.506)	(0.502)	(0.499)
N	395	394	393	197	196	195

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The table displays the statistics for the GARCH(1,1) model from Section 2.6 of the main text for the training period (1990:01 - 2006:06) and the for the full-sample period (1990:01 - 2022:12)). As highlighted in the main text, the GARCH model is recursively estimated each period after the training period to be free from look-ahead bias for the second half of the sample period (the period from July, 2006, through December, 2022). The full sample and training sample results shown here provide a snapshot of the performance of the GARCH model at two points in time and demonstrate that the estimated coefficients are relatively stable. *pd* is the price-dividend ratio on the S&P 500 index from Robert Shiller's website. Returns are in percent. Standard errors are in parentheses.

Figure 3. Equity Premium (Approximation versus Exact)



Notes: The figure displays the exact equity premium in the NEO-EU model calculated according to Equation 23:

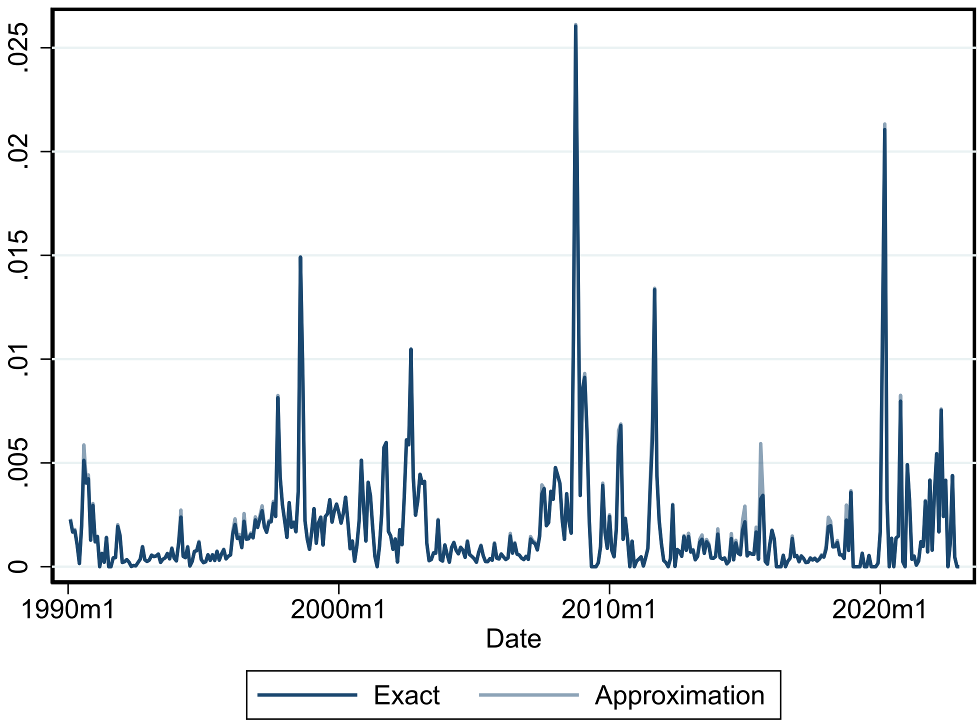
$$E_t R_{t+1} - R_{f,t} = e^{\mu_t + \frac{1}{2}q_t^2}\left(1 - e^{-\eta\lambda\sigma q_t}\right) + \frac{\gamma_t \alpha_t}{1-\gamma_t}\left(e^{-\lambda\bar{\xi}\sigma - \frac{1}{2}\lambda^2\sigma^2}\left(R_{f,t} - e^{\mu_t + \bar{\xi} q_t}\right) + (1-\alpha_t)e^{\lambda\underline{\xi}\sigma - \frac{1}{2}\lambda^2\sigma^2}\left(R_{f,t} - e^{\mu_t - \underline{\xi} q_t}\right)\right),$$

and the approximation to the equity premium calculated according to Equation 10:

$$E_t R_{t+1} - R_{f,t} \approx \big(1 - 2\alpha_t + \lambda\sigma\xi\big)\xi\gamma_t q_t.$$

Both equations use coefficient of relative risk aversion $\lambda = 1$ (log utility). The exact equity premium is shown in dark blue. The approximation is shown in light blue. The correlation between the two series is 0.999.

Figure 4. Variance Risk Premium (Approximation versus Exact)



Notes: The figure displays the exact variance risk premium in the NEO-EU model according to Equation 26:

$$VRP_t = (1-\gamma_t)E_t R_{f,t} M_{t+1}(R_{t+1} - R_{f,t})^2 + \gamma_t \Big(\alpha_t R_{f,t} \overline{M}_{t+1}(\overline{R}_{t+1} - R_{f,t})^2 + (1-\alpha_t) R_{f,t} \underline{M}_{t+1}(\underline{R}_{t+1} - R_{f,t})^2\Big) - q_t^2,$$

and the approximation to the variance risk premium calculated according to Equation 12:

$$VRP_t \approx \gamma_t q_t^2 (\xi^2 - 1).$$

The exact variance risk premium is shown in dark blue. The approximation is shown in light blue. The correlation between the two series is 0.999.

Appendix C Robustness Appendix

This appendix contains the results of our robustness tests. Section C.1 conducts the stability tests, subsample tests, and additional out-of-sample tests using alternative GARCH volatility models to study the risk-return tradeoff. Section C.2 tests the ability of α to predict returns by itself. Section C.3 tests the risk-return tradeoff over the longer six-month and twelve-month horizons. Section C.4 tests the log-linearized version of Equation (11) from the main text. Section C.5 tests if our results for predicting crashes and recessions holds for the out-of-sample period. Section C.6 tests the risk-return tradeoff in-sample and out-of-sample for the α series constructed from the equity premium approximation in Section 2 with risk aversion, using CRRA parameter $\lambda = 1$ (log utility). The results with $\lambda = 2$ (the level of risk aversion used in the investment application) are very similar.

C.1 Alternative GARCH Volatility Models

We test if the risk-return tradeoff results also hold if q_t is constructed as a standard simple GARCH(1,1) model (without including the price-dividend ratio) or as a GJR GARCH model. As with our main specification (the GARCH(1,1) model from Section 2.6), both the simple GARCH(1,1) model and the GJR GARCH model are recursively estimated and are free from look-ahead bias.

Table 14 shows the risk-return tradeoff results for the case where q_t is a standard simple GARCH(1,1) model (similar to that in Section 2.6 but constructed with a constant mean instead of a time-varying mean based on the price-dividend ratio). The table shows the results for both monthly and quarterly forecast horizons and for the full sample and each subsample.

The results for the standard simple GARCH model are similar to our baseline results: By itself, q_t does not predict the equity premium, but including both q_t and $\alpha_t q_t$ yields a positive and significant coefficient on q_t and a negative and significant coefficient on $\alpha_t q_t$ at both the monthly and quarterly forecast horizon. At the monthly horizon, the R^2 jumps from 0.4% with only q_t to 4% with both q_t and $\alpha_t q_t$. At the quarterly horizon, the R^2 jumps from 1.1% with only q_t to 10.3% with both q_t and $\alpha_t q_t$. These results for the full sample are stronger for the out-of-sample period. For the training period, the coefficients for both the monthly and quarterly horizon forecasts are also significant at the 10% level for q_t and at the 5% level for $\alpha_t q_t$.

Table 13 reveals that the coefficient estimates under the simple GARCH model are similar across the two halves of the sample period when both q_t and $\alpha_t q_t$ are included in the regression. For example, for q_t, the estimated coefficient is 1.64 for the first half and 1.41 for the second half at the monthly horizon. For $\alpha_t q_t$, the estimated coefficient is -1.34 for the first half and -1.67 for the second half of the sample at the monthly horizon. In contrast, including q_t by itself in the regression produces unstable estimates that change from negative to positive across the two halves of the sample at both the monthly and quarterly horizons.

Figure 5 displays the predictive performance over time (the plots of the difference in cumulative sum of squared errors, $\Delta CSSE$) for out-of-sample regressions with q_t (left panel) and both q_t and $\alpha_t q_t$ (right panel) for the one-month forecast horizon where q_t is the simple GARCH volatility. While by itself, q_t under-performs the benchmark, the forecast with both q_t and $\alpha_t q_t$ consistently outperforms the benchmark with a $\Delta CSSE$ above 1% that increases across the out-of-sample period and is close to 2% by the end of the sample period.

In addition to the simple GARCH(1,1) model, we apply a GJR GARCH model. The results for the GJR GARCH model are similar to those for the simple GARCH model. Figure 6 shows the out-of-sample performance of the GJR GARCH model over time which is similar to that of the simple GARCH model shown in Figure 5.

Table 13. Stability of Coefficients in Predictive Regressions (Simple Model)

		Monthly		Quarterly	
x_t	z_t	β_1	β_2	β_1	β_2
q_t		-0.05	0.43	-0.67	1.51
q_t	$\alpha_t q_t$	1.64	1.41	3.86	4.09
$\alpha_t q_t$	q_t	-1.34	-1.67	-4.00	-4.34

Notes: The table displays coefficients from full-sample predictability regressions for the first and second half of the sample period for the monthly and quarterly forecast horizons. The first sub-sample spans monthly data from 1990:01 - 2006:06. The second sub-sample spans from 2006:07 - 2022:12. β_1 and β_2 denote the estimated coefficients for the first and second halves of the sample. They are estimated from the regression $r^e_{[t+1,t+h]} = a + \beta\, x_t + \beta_D\, D + \beta_{Dx}\, D\, x_t + \beta_z z_t + \epsilon_{[t+1,t+h]}$ where D is a dummy variable that equals 0 in the first half of the sample period and 1 in the second half of the sample, $h \in \{1, 3\}$, $\beta_1 := \beta$ and $\beta_2 := \beta + \beta_{Dx}$. The predictor variables include market ambiguity attitude, α_t, conditional market volatility, q_t, measured from a simple GARCH(1,1) model, and the product $\alpha_t q_t$. For ease of interpreting the coefficients, q_t and $\alpha_t q_t$ are divided by their (full sample) standard deviation.

Table 14. Market Ambiguity Attitude and the Risk-Return Tradeoff (Simple GARCH Volatility)

Monthly	Full Sample (1990 - 2022)			Out-of-Sample Period			Training Sample Period		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}
q_t	0.28		1.34***	0.42		1.53***	-0.04		1.34*
	(1.09)		(5.26)	(1.35)		(5.22)	(-0.09)		(1.93)
$\alpha_t q_t$		-0.32	-1.36***		-0.28	-1.71**		-0.35	-1.24**
		(-1.07)	(-3.73)		(-0.55)	(-2.59)		(-1.01)	(-2.06)
R^2	0.004	0.005	0.040	0.011	0.003	0.060	0.000	0.007	0.023
Quarterly	Full Sample (1990 - 2022)			Out-of-Sample Period			Training Sample Period		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel B	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$
q_t	0.85		3.83***	1.50**		4.48***	-0.61		2.97*
	(1.24)		(5.59)	(1.99)		(5.69)	(-0.50)		(1.81)
$\alpha_t q_t$		-0.90	-3.86***		-0.50	-4.67***		-1.23	-3.21**
		(-1.15)	(-3.95)		(-0.39)	(-2.89)		(-1.28)	(-2.09)
R^2	0.011	0.010	0.103	0.044	0.003	0.164	0.004	0.029	0.054

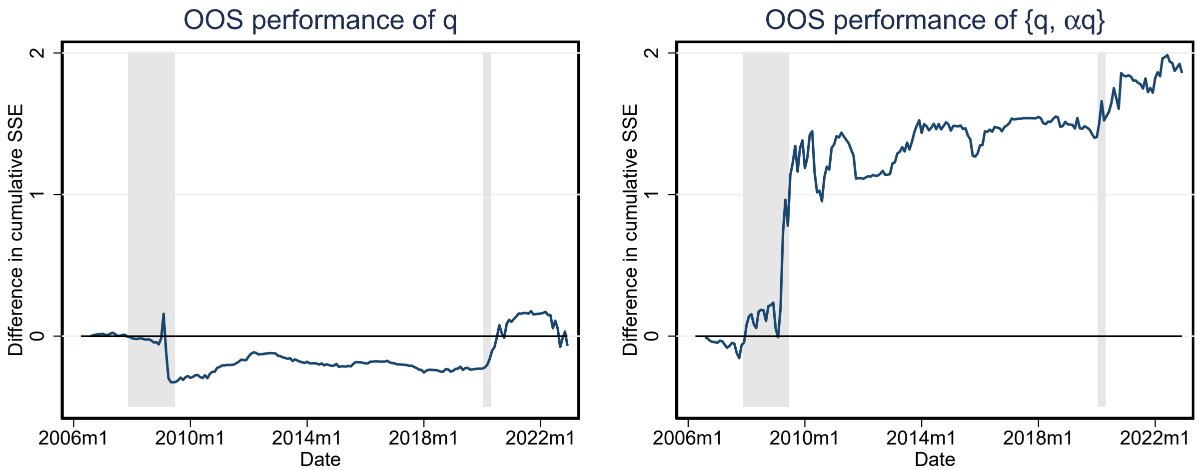
Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium, $r^e_{[t+1,t+h]}$, in percent, against the conditional stock market volatility (q_t), estimated from a standard simple GARCH(1,1) model, in regression specifications (1), (4), and (7); regressions of the log equity premium against the product of market ambiguity attitude and the conditional market volatility ($\alpha_t q_t$) in regression specifications (2), (5), and (8); and regressions of the log equity premium against both variables in regression specifications (3), (6), and (9). Formally, we run versions of the following regression that include one or both of the right-hand-side variables:

$$r^e_{[t+1,t+h]} = \beta_0 + \beta_1 q_t + \beta_2 \alpha_t q_t + \epsilon_{[t+1,t+h]}. \quad (29)$$

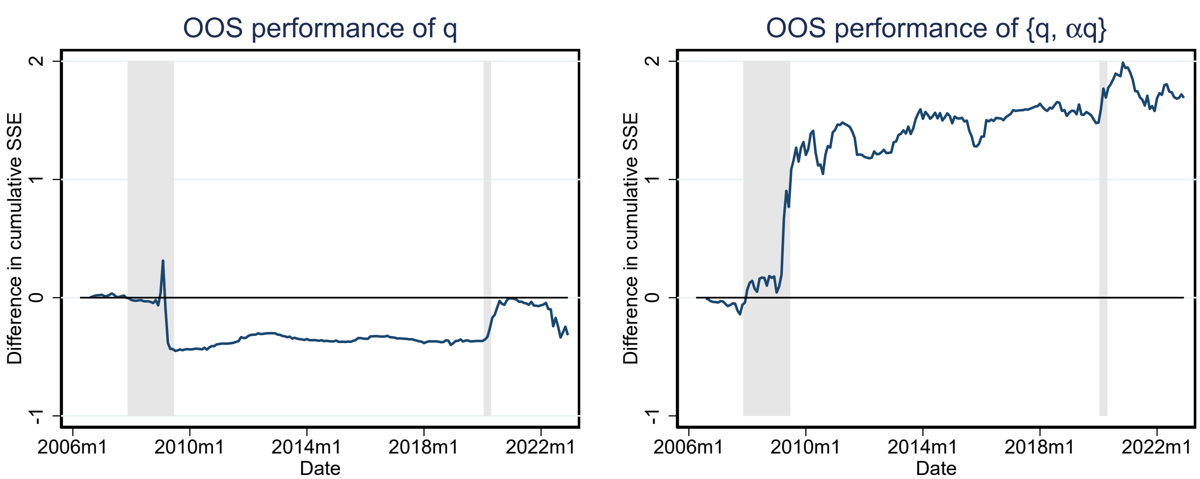
Regressions (1), (2), and (3) span monthly data from the full sample period (1990:01 - 2022:12). Regressions (4), (5), and (6) use data from the second half of this sample (2006:07 - 2022:12) which is the period for which q_t and α_t are recursively estimated using only information available to investors in real time. Regressions (7), (8), and (9) use data from the first half of the sample period (1990:01 - 2006:06) which served as the training period in which q_t and α_t were estimated using all data in the first half of the sample. Regressions in Panel A are over a forecast horizon of $h = 1$ month (monthly horizon). Regressions in Panel B are over a forecast horizon of $h = 3$ months (quarterly horizon) and the dependent variable is the cumulative three-month log equity premium. For ease of interpreting the coefficients, q and αq are divided by their (full sample) standard deviation.

Figure 5. The Risk-Return Tradeoff Out-of-Sample with α and Simple GARCH Volatility



Notes: This figure displays the difference in the cumulative sum of squared errors, $\Delta CSSE_{OOS}$, between the forecast of the one-month-ahead log equity premium based on the historical average and the forecast based on the conditional market volatility from a standard simple GARCH(1,1) model in the left panel. The right panel displays the $\Delta CSSE_{OOS}$ between the forecast based on the historical average and the forecast based on the pair of predictors consisting of the conditional market volatility and the product of the conditional market volatility and the conditional market ambiguity attitude. The out-of-sample period spans from 2006:07 - 2022:12. Shaded periods are NBER recessions.

Figure 6. The Risk-Return Tradeoff Out-of-Sample with α and GJR GARCH Volatility



Notes: This figure displays the difference in the cumulative sum of squared errors, $\Delta CSSE_{OOS}$, between the forecast of the one-month-ahead log equity premium based on the historical average and the forecast based on the conditional market volatility from a standard GJR GARCH model in the left panel. The right panel displays the $\Delta CSSE_{OOS}$ between the forecast based on the historical average and the forecast based on the pair of predictors consisting of the conditional market volatility and the product of the conditional market volatility and the conditional market ambiguity attitude. The out-of-sample period spans from 2006:07 - 2022:12. Shaded periods are NBER recessions.

Table 15 displays the R^2_{OS} statistic for out-of-sample forecasts using the simple GARCH volatility model (top panel) and using the GJR GARCH volatility model (bottom panel) for the monthly, quarterly, six-month, and annual forecast horizons. Results are similar to the out-of-sample results shown in the main text.

Table 15. R^2_{OS} (percent) for Log Equity Premium Forecasts

Simple GARCH	Monthly Horizon		Quarterly Horizon		Six-Month Horizon		Annual Horizon	
Predictors	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW
q_t	-0.16	-0.50	-1.77	-1.21	-2.91	-2.38**	-7.73	-3.84***
q_t, $\alpha_t q_t$	4.21	2.63***	12.77	4.26***	19.09	4.41***	14.44	4.71***
GJR GARCH	**Monthly Horizon**		**Quarterly Horizon**		**Six-Month Horizon**		**Annual Horizon**	
Predictors	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW
q_t	-0.73	-0.71	-3.98	-1.54	-4.46	-2.57**	-9.61	-3.83***
q_t, $\alpha_t q_t$	3.83	2.63***	10.66	4.08***	18.67	4.05***	14.61	4.15***

Notes: The Table displays the Campbell and Thompson (2008) R^2_{OS} statistic (in percent) for predictor variables at the monthly, quarterly, six-month, and annual (twelve-month) forecast horizons of the log equity premium. The dependent variable is, respectively, the one-month, cumulative three-month, cumulative six-month, and cumulative twelve-month log equity premium. The sets of predictors are market volatility (q_t), and market volatility and the product of volatility and ambiguity attitude (q_t, $\alpha_t q_t$). The top panel shows the results for which volatility q is generated by a simple GARCH(1,1) model. The bottom panel shows the results for which q is generated by a GJR GARCH model. CW is the Clark and West (2007) MSPE-adjusted statistic. **, and *** denotes significance at the 5%, and 1% levels. The out-of-sample period spans the second half of our sample, 2006:07 - 2022:12.

C.2 Market Ambiguity Attitude without Market Volatility

Table 16 reports predictive regressions with α_t as the predictor variable at the monthly and quarterly forecast horizons. The table shows that α_t itself has predictive power at both horizons, with an adjusted R^2 of 1.6% at the monthly horizon and 4.6% at the quarterly horizon. Figure 7 shows the out-of-sample prediction performance of market volatility, q, market ambiguity attitude, α, the combination of q and αq, and for comparison, short interest, which is among the strongest known predictors of aggregate stock returns (Rapach et al., 2016). Each of the forecasts except for q show evidence of positive predictability. Only the bivariate forecast of q and αq consistently has a difference in the cumulative sum of squared errors around 1.5% and displays an upward slope.

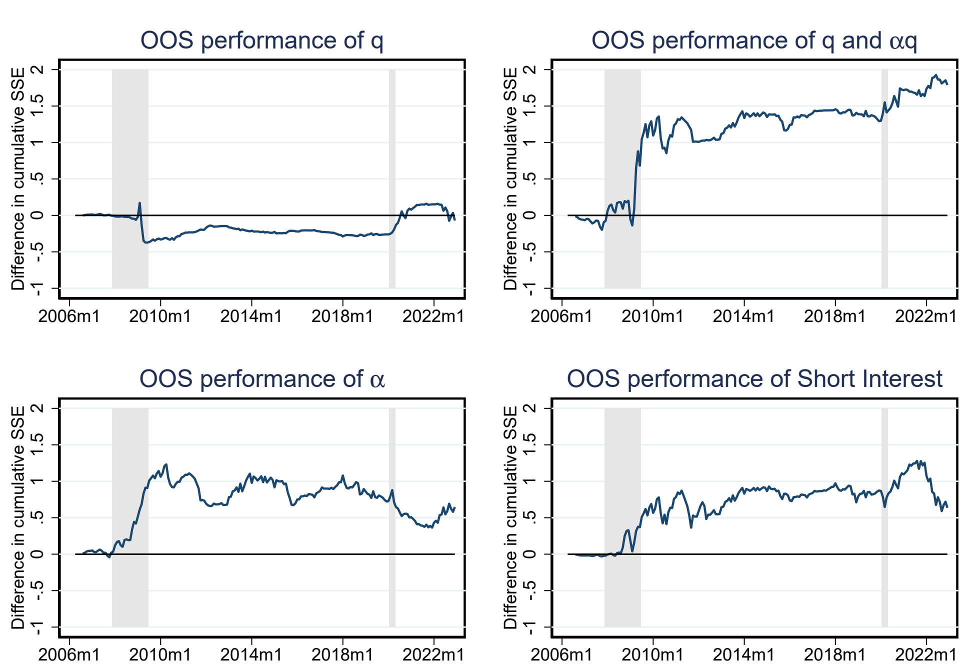
Table 16. Predictive Regressions with α

	Monthly		Quarterly
	(1)		(2)
	r^e_{t+1}		$r^e_{[t+1,t+3]}$
α_t	-0.57**	α_t	-1.67***
	(-2.16)		(-2.33)
adj. R^2	0.016		0.046

** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium, $r^e_{[t+1,t+h]}$, (in percent) against market ambiguity attitude, α_t. In the regression specifications in columns (1) and (2), the dependent variable is the one-month and cumulative three-month log equity premium. For ease of interpreting the coefficients, α_t is divided by its (full sample) standard deviation. The sample period spans monthly data from 1990:01 through 2022:12. Newey-West t statistics are in parentheses.

Figure 7. Out-of-Sample Equity Premium Prediction



Notes: This figure displays the difference in cumulative sum of squared errors, $\Delta CSSE_{OOS}$, between the one-month-ahead forecast of the log equity premium based on the historical average and the one-month-ahead forecast based on the conditional market volatility, q, the conditional ambiguity attitude, α, the combination of q and αq, and the leading equity premium predictor, short interest. The out-of-sample period spans the second half of the sample period, 2006:07 - 2022:12. Shaded periods are NBER recessions.

C.3 Long-Horizon Regressions

Table 17 conducts predictive regressions with q_t and $\alpha_t q_t$ at the longer six-month and twelve-month (annual) horizons. The results reinforce the strong complementary predictive power of $\alpha_t q_t$ that is documented in the main text. At the six-month horizon, including $\alpha_t q_t$ in the regression raises the adjusted R^2 dramatically from 2.7% to 17.8%. Similar results hold for the annual horizon.

Table 17. The Risk-Return Tradeoff over Long Horizons

	Six-Month Horizon			Annual Horizon	
	(1) $r^e_{[t+1,t+6]}$	(2) $r^e_{[t+1,t+6]}$		(3) $r^e_{[t+1,t+12]}$	(4) $r^e_{[t+1,t+12]}$
q_t	1.97*	7.27***	q_t	2.98	11.79***
	(1.73)	(4.81)		(1.49)	(3.31)
$\alpha_t q_t$		-6.95***	$\alpha_t q_t$		-11.56***
		(-3.65)			(-2.94)
adj. R^2	0.027	0.178		0.030	0.231

Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium, $r^e_{[t,t+h]}$, (in percent) against market volatility, q_t. Even-numbered regressions also include $\alpha_t q_t$. In the regression specifications in columns (1) and (2), the dependent variable is the cumulative six-month log equity premium and the predictor variables are lagged by six months (semi-annual forecast horizon). In the regression specifications in columns (3) and (4), the dependent variable is the cumulative twelve-month log equity premium and all predictors are lagged twelve months (annual forecast horizon). For ease of interpreting the coefficients, q_t and $\alpha_t q_t$ are divided by their (full sample) standard deviation. The sample period spans monthly data from 1990:01 through 2022:12.

C.4 Log-linearized Model

Log-linearizing the equity premium formula in Equation (11) yields an approximation of the log equity premium as a linear function of market volatility, q_t, market ambiguity, γ_t, and market optimism, α_t. Table 18 reports the results of predictive regressions against these lagged state variables. Consistent with our main results, α_t restores the risk-return tradeoff at the monthly, quarterly, six-month, and annual horizons. The state variable, γ_t plays less of a role, but the coefficent for γ_t is positive and significant at the six-month horizon, and including γ_t in the regression at that horizon increases the adjusted R^2 by 3.2%.

Table 18. Predictive Regressions with Log-Linearized Model

	Monthly Horizon		Quarterly Horizon		Six-Month Horizon		Annual Horizon	
	(1) r^e_{t+1}	(2) r^e_{t+1}	(3) $r^e_{[t+1,t+3]}$	(4) $r^e_{[t+1,t+3]}$	(5) $r^e_{[t+1,t+6]}$	(6) $r^e_{[t+1,t+6]}$	(7) $r^e_{[t+1,t+12]}$	(8) $r^e_{[t+1,t+12]}$
α_t	-0.98***	-1.00***	-2.87***	-3.27***	-5.60***	-6.31***	-9.60***	-10.34***
	(-4.14)	(-4.14)	(-4.46)	(-4.92)	(-4.11)	(-4.64)	(-3.29)	(-3.63)
q_t	0.80***	0.83***	2.39***	2.81***	4.82***	5.56***	7.83***	8.62***
	(3.29)	(2.63)	(4.14)	(3.33)	(5.76)	(5.36)	(3.64)	(3.90)
γ_t		0.07		1.23		2.20**		2.37
		(0.19)		(1.65)		(2.13)		(1.49)
adj. R^2	0.035	0.033	0.110	0.130	0.208	0.240	0.285	0.302

Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium, $R^e_{[t,t+h]}$, (in percent) against lagged market ambiguity attitude, α_t and market volatility, q_t. Even-numbered regressions also include lagged γ_t. For ease of interpreting the coefficients, q_t, α_t, and γ_t are divided by their (full sample) standard deviation. The sample period spans monthly data from 1990:01 through 2022:12.

C.5 Predicting Market Crashes and Recessions in the Out-of-Sample Period

Table 19 tests if our results for market crashes hold for the out-of-sample period during which α_t is free from look-ahead bias. The table reveals that α_t predicts both market corrections (10% crashes) as well as 5% crashes in the out-of-sample period. The predictability holds even controlling for the variables used in the construction of α_t (q_t, VIX, and the price-dividend ratio), which themselves are natural candidates for predicting crashes.

Table 19. Predicting Market Crashes with Market Ambiguity Attitude

	Logistic Regressions for Predicting Market Crashes (Out-of-Sample Period)									
	(1) -10%	(2) -10%	(3) -10%	(4) -10%	(5) -10%	(6) -5%	(7) -5%	(8) -5%	(9) -5%	(10) -5%
α_{t-3}	1.90***	2.11**	1.68***	1.85***	2.17***	0.76***	0.81***	0.80***	0.77***	0.86***
	(3.49)	(2.39)	(5.25)	(4.01)	(3.94)	(2.94)	(3.14)	(2.98)	(2.86)	(3.00)
q_{t-3}		0.54			0.12		-0.14			-0.17
		(1.20)			(0.31)		(-0.73)			(-0.83)
VIX_{t-3}			0.35		-0.39			-0.08		-0.03
			(0.82)		(-0.78)			(-0.31)		(-0.12)
pd_{t-3}				-1.60*	-2.21				-0.04	-0.17
				(-1.84)	(-1.37)				(-0.14)	(-0.45)
Pseudo R^2	0.159	0.214	0.188	0.271	0.284	0.072	0.076	0.073	0.072	0.077

Robust Z statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays the slope coefficients from logistic regressions. The left-hand-side variable equals one in period t if a market return less than -10% occurred in period t, and zero otherwise. In columns (6) - (10), the left-hand-side variable equals one in period t if a market return less than -5% occurred in period t, and zero otherwise. The right-hand-side variables (each lagged three months) are the market ambiguity attitude. α, the conditional market volatility, q, the VIX index of the Chicago Board of Options Exchange, and the price-dividend ratio, pd, of the S&P 500 index. The results are shown for the out-of-sample period (2006:07 - 2022:12). For convenience in interpreting the coefficients, each right-hand-side variables is divided by its full-sample standard deviation.

Table 20 summarizes logistic regressions with α as a predictor variable for NBER recessions at the three-month horizon during the out-of-sample period.[18] Recall that we consider the term spread, TMS, the aggregate stock market return, R_m, and the default yield spread (DFY) as candidate NBER recession predictors. Each of these variables has significant predictive power for NBER recessions over our sample period. As additional control variables we include the price dividend ratio of the S&P 500 index (pd), the VIX index from the CBOE, and q from the GARCH(1,1)

[18] Similar results are obtained using probit regressions.

model in Section 2.6. We also include as controls the Baker and Wurgler (2006) market sentiment index and the lagged NBER recession indicator.

Table 20 shows that α significantly predicts NBER recessions in the out-of-sample period across each set of control variables. For regression specification (6) with all eight control variables included, adding α to the regression increases the Pseudo R^2 by 14.7 percentage points. The results in this section further indicate that market ambiguity attitude predicts stock market fluctuations and business cycle fluctuations.

Table 20. Predicting Recessions with Market Ambiguity Attitude

Logistic Regressions for Predicting Recessions (Out-of-Sample Period)

	(1) REC	(2) REC	(3) REC	(4) REC	(5) REC	(6) REC
α_{t-3}	2.27***	3.57***	2.10***	3.00***	2.77***	3.25***
	(3.02)	(3.55)	(3.14)	(4.44)	(3.94)	(3.26)
Lagged Recession Predictors	NO	YES	NO	NO	NO	YES
Lagged Recession Indicator	NO	NO	YES	NO	NO	YES
Lagged α Ingredients	NO	NO	NO	YES	NO	YES
Lagged Sentiment Index	NO	NO	NO	NO	YES	YES
Pseudo R^2	0.290	0.593	0.617	0.612	0.379	0.747
Δ(Pseudo R^2)	0.290	0.275	0.142	0.311	0.375	0.147

Robust z statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays the slope coefficients from logistic regressions. The left-hand-side variable is the NBER recession indicator (REC) from the St. Louis Federal Reserve database. REC is equal to one in period t if there was a recession in period t and is equal to zero otherwise. The right-hand-side variables (each lagged three months) include the market ambiguity attitude (α) and eight control variables: (i) the term spread (TMS) (the difference between the long-term U.S. government bond yield and the U.S. treasury bill) from Welch and Goyal (2008) (ii) the default yield spread (DFY) (the difference between BAA and AAA-rated corporate bond yields) from Welch and Goyal (2008) (iii) the aggregate market return (R_m) from Kenneth French's data library (The variables TMS, DFY, and R_m are our 'recession predictor' variables as these are known to have forecasting power for NBER recessions); (iv) the lagged NBER recession indicator; (v) the price-dividend ratio of the S&P 500 index from Robert Shiller's website (pd); (vi) the VIX index of the Chicago Board of Options Exchange; (vii) the conditional market volatility, q, from the GARCH model in Section 2.6 (The variables pd, VIX, and q are the 'α ingredients' as these variables were used in the construction of α); and (viii) the Baker and Wurgler (2006) market sentiment index. The table displays the results for the out-of-sample period from 2006:07 through 2022:12, except in regression specification (2) which ends in 2021:12, specification (5) which ends in 2022:06, and specification (6) which ends in 2021:12 due to data availability. Δ(Pseudo R^2) denotes the change in Pseudo R^2 from including α in the regression relative to an otherwise identical regression that excludes α. For convenience in interpreting the coefficients, α is divided by its full-sample standard deviation.

C.6 Market Ambiguity Attitude accounting for Risk Aversion

As in the main text, we use the square of the VIX index as a proxy for the risk-neutral variance, $\text{Var}_t^Q R_{t+1}$. Then using formula (12) we find γ_t to be:

$$\hat{\gamma}_t \approx \frac{1}{\xi^2 - 1}\Big(\frac{\text{VIX}_t^2}{\hat{q}_t^2} - 1\Big). \tag{30}$$

Next, we use the relationship, $\text{EP}_t \approx \xi(1 - 2\alpha_t + \lambda\sigma\xi)q_t\gamma_t$ from Equation (10) to estimate α_t. In line with the intuition that α_t has persistent dynamics, we let α_t follow a Markov-switching structure with two states. Note that the relationship implies $\xi(1 - 2\alpha_t + \lambda\sigma\xi) \approx \frac{\text{EP}_t}{q_t\gamma_t}$. Thus, if α_t follows a Markov-switching model, so does the ratio $\frac{\text{EP}_t}{q_t\gamma_t}$. To estimate α_t, we estimate the following Markov-switching dynamic regression model:

$$\frac{\hat{\text{EP}}_t}{\hat{q}_t\hat{\gamma}_t} = \mu_{m_t} + \epsilon_t, \tag{31}$$

where ϵ_t is a white noise and μ_{m_t} switches between two regimes according to a probability matrix.

The quantity $\frac{\text{EP}_t}{q_t\gamma_t}$ is a measure like a conditional Sharpe ratio but which includes a role for market ambiguity, γ_t. In the Markov-Switching model there are two regimes: (i) a bear market regime with relatively low prices and high expected future returns per unit of risk, and (ii) a bull market regime with relatively high prices and low expected future returns per unit of risk. Market optimism, α_t, is then increasing in the probability of the bull market regime.

The estimated model gives us a predicted value of $\hat{\mu}_{m_t}$ using the information up to and including time t. We then find our estimate of α_t according to:

$$\hat{\alpha}_t = \frac{1}{2}\Big(1 - \frac{\hat{\mu}_{m_t}}{\xi} + \lambda\sigma\xi\Big). \tag{32}$$

As before both q_t and α_t are estimated dynamically so that *only* the information up to period t is used in the estimation of $\hat{q}_t$ and $\hat{\alpha}_t$ to avoid look-ahead bias.

For the standard deviation of consumption growth we use the monthly value data value, noted in the proof of Proposition 1 from the main text, given by $\sigma = 0.016/\sqrt{12}$ and we use $\xi = 4.77$ as before.

We construct two alternative α_t series, denoted $\alpha_{\lambda,t}$ in which $\lambda = 1$ (log utility) and $\lambda = 2$ (the level of risk aversion in our investment application in the main text). The baseline α_t series has a correlation of 0.999 with each of these two new series. The mean value of α_t changes slightly from 0.27 (with $\lambda = 0$) to 0.28 (with $\lambda = 1$) to 0.29 (with $\lambda = 2$).

Table 21 reports the R^2_{OS} statistic for predictive regressions at the monthly, quarterly, six-month, and annual horizons using α_t constructed for a representative agent with log utility ($\lambda = 1$). Table 22 displays the in-sample predictive regressions for the full sample period and for both sub-samples using $\alpha_{\lambda,t}$ with $\lambda = 1$. The results are close to those for our baseline α_t series in which the representative agent is risk-neutral ($\lambda = 0$). The results for the α_t series constructed with $\lambda = 2$ are also similar and are omitted.

Table 21. R^2_{OS} for the Risk-Return Tradeoff

	Monthly Horizon		Quarterly Horizon		Six-Month Horizon		Annual Horizon	
Predictors	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW	R^2_{OS}	CW
q_t, $\alpha_{\lambda,t}q_t$	3.92	2.53**	12.52	4.01***	19.00	4.20***	11.00	4.52***

Notes: The table displays the Campbell and Thompson (2008) R^2_{OS} statistic (in percent) for predictor variables at the monthly, quarterly, six-month, and annual (twelve-month) forecast horizons of the log equity premium. The dependent variable is, respectively, the one-month, cumulative three-month, cumulative six-month, and cumulative twelve-month log equity premium. The set of predictors is market volatility (q_t) and the product of volatility and ambiguity attitude (q_t, $\alpha_{\lambda,t}q_t$), where $\alpha_{\lambda,t}$ is measured from the equity premium approximation in which the representative agent has log utility ($\lambda = 1$). CW is the Clark and West (2007) MSPE-adjusted statistic. ** and *** denotes significance at the 5%, and 1% levels. The out-of-sample period spans 2006:07 - 2022:12.

Table 22. Market Ambiguity Attitude and the Risk-Return Tradeoff with Risk Aversion

Monthly	Full Sample (1990 - 2022)			Out-of-Sample Period			Training Sample Period		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}	r^e_{t+1}
q_t	0.30		1.36***	0.43		1.52***	-0.02		1.59**
	(1.18)		(5.46)	(1.38)		(5.57)	(-0.04)		(2.29)
$\alpha_{\lambda,t} q_t$		-0.30	-1.37***		-0.25	-1.68***		-0.35	-1.39**
		(-1.03)	(-3.91)		(-0.50)	(-2.63)		(-1.01)	(-2.35)
R^2	0.005	0.005	0.041	0.011	0.003	0.059	0.000	0.007	0.027

Quarterly	Full Sample (1990 - 2022)			Out-of-Sample Period			Training Sample Period		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel B	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$	$r^e_{[t+1,t+3]}$
q_t	0.92		3.94***	1.50**		4.46***	-0.51		3.71**
	(1.38)		(5.65)	(2.02)		(5.82)	(-0.40)		(2.29)
$\alpha_{\lambda,t} q_t$		-0.84	-3.89***		-0.42	-4.60***		-1.22	-3.65**
		(-1.09)	(-4.09)		(-0.34)	(-2.93)		(-1.24)	(-2.42)
R^2	0.014	0.011	0.111	0.046	0.002	0.163	0.003	0.027	0.062

Newey-West t statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The table displays regressions of the log equity premium, $r^e_{[t+1,t+h]}$, in percent, against the conditional stock market volatility (q_t), estimated from a GARCH(1,1) model (from Section 2.6), in regression specifications (1), (4), and (7); regressions of the log equity premium against the product of market ambiguity attitude and the conditional market volatility ($\alpha_{\lambda,t} q_t$) in regression specifications (2), (5), and (8); and regressions of the log equity premium against both variables in regression specifications (3), (6), and (9). Market ambiguity attitude $\alpha_{\lambda,t}$ is measured from the equity premium approximation in which the representative agent has log utility ($\lambda = 1$). Formally, we run versions of the following regression that include one or both of the right-hand-side variables:

$$r^e_{[t+1,t+h]} = \beta_0 + \beta_1 q_t + \beta_2 \alpha_{\lambda,t} q_t + \epsilon_{[t+1,t+h]}. \tag{33}$$

Regressions (1), (2), and (3) span monthly data from the full sample period (1990:01 - 2022:12). Regressions (4), (5), and (6) use data from the second half of this sample (2006:07 - 2022:12) which is the period for which q_t and α_t are recursively estimated using only information available to investors in real time. Regressions (7), (8), and (9) use data from the first half of the sample period (1990:01 - 2006:06) which served as the training period in which q_t and α_t were estimated using all data in the first half of the sample. Regressions in Panel A are over a forecast horizon of $h = 1$ month (monthly horizon). Regressions in Panel B are over a forecast horizon of $h = 3$ months (quarterly horizon) and the dependent variable is the cumulative three-month log equity premium. For ease of interpreting the coefficients, q and αq are divided by their (full sample) standard deviation.

Appendix D Expected Log Market Return and the PD Ratio

LEMMA 1. *If the payoff X_{t+1} in the model is replaced with the dividend D_{t+1}, then replacing Assumption 1 with $\overline{D}_{t+1} = (1+\xi q_t)D_t$, $\underline{D}_{t+1} = (1-\xi q_t)D_t$, and $E_t[D_{t+1}] = D_t$, gives approximately the same equation for the equity premium in (11). Moreover, the expected log return is approximately linear in the price-dividend ratio.*

Proof. Given the best and worst case scenarios for future dividend, i.e., $\overline{D}_{t+1} = (1 + \xi q_t)D_t$ and $\underline{D}_{t+1} = (1 - \xi q_t)D_t$, the price Equation (5) for $\lambda = 0$ becomes

$$P_t = (1 - \gamma_t)\delta E_t[D_{t+1}] + \gamma_t\delta\big(\alpha_t\overline{D}_{t+1} + (1 - \alpha_t)\underline{D}_{t+1}\big)$$

$$P_t = (1 - \gamma_t)\delta D_t + \gamma_t\delta\Big(\alpha_t(1 + \xi q_t)D_t + (1 - \alpha_t)(1 - \xi q_t)D_t\Big).$$

Thus, $log(pd_t) = log(\delta) + log\big(1 - \xi\gamma_t q_t(1 - 2\alpha_t)\big)$, and using the approximation $log(1 + x) \approx x$, we find that as is the case with pd_t, the log price-dividend ratio $log(pd_t)$ is (approximately) linear in $\xi\gamma_t q_t(1 - 2\alpha_t)$. Note that the approximation is accurate if $\gamma_t q_t$ is small, and in our monthly data, both γ_t (on average 0.05) and q_t (on average 0.04) are small.

As for the expected return, we have $E_t R_{t+1} = \frac{E_t[D_{t+1}]}{P_t} = \frac{1}{pd_t}$, so the log expected return is linear in $log(pd_t)$, which is linear $\xi\gamma_t q_t(1 - 2\alpha_t)$. Thus, we showed that approximately, both pd_t and the log expected return are linear in $\xi\gamma_t q_t(1 - 2\alpha_t)$, and hence, the log expected return is also linear in pd_t. Finally, note that the log expected return and expected log return are off by a Jensen's term. Thus, as long as this Jensen's term is negligible, the expected log return is approximately linear in the price-dividend ratio. The last condition on the negligibility of Jensen's term can be checked in the data, and we find that in our monthly data, log expected returns and expected log returns have an almost perfect correlation.

□